Calculating high-frequency resistive losses in wires

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This tutorial serves two functions:

Document some useful general results on the effective resistance and ohmic losses in wires at high frequency.

Show how to set up calculations of magnetic field distributions in wires with the two-dimensional code **Nelson**.

To start, let's review some basic equations for a long wire with cross-section area A_w (m^2) and conductivity σ (S/m). At low frequency, the current density is uniformly distributed across the wire and the resistance per length is

$$R = \frac{1}{\sigma A_w}. \quad (\Omega/\mathrm{m}) \tag{1}$$

If the wire carries an alternating current $I_0 \cos(2\pi f t)$ at frequency f, the time-averaged power dissipation is

$$\overline{P} = \frac{I_0^2 R}{2}. \quad (\mathrm{W/m}) \tag{2}$$

At high values of f, the magnetic field associated with the current flow is inhibited from penetrating the metal wire, concentrating the current density near the wire surface. The effective effective cross-section area of the wire is therefore smaller than A_w, increasing the resistance per length and the ohmic power loss for a given current. The penetration distance for magnetic fields, called the *skin depth*, is given by

$$\delta = \sqrt{\frac{1}{\pi \mu \sigma f}} \quad (\mathrm{m}). \tag{3}$$

The quantity μ is the magnetic permeability, given by

$$\mu = (1.257 \times 10^{-6})\ \mu_r, \tag{4}$$

where μ_r is the relative magnetic permeability. The relative permeability equals 1.0 for most wire metals.

The skin depth allows us to define the high and low-frequency regimes. For a circular wire of radius r_w, enhanced ohmic losses occur when

$$\delta \leq r_w. \tag{5}$$

In the following calculation, we'll use the example of a circular copper wire 1.0 mm in radius. The conductivity of copper is $\sigma = 5.814 \times 10^7$ S/m. Using Eq. 3, the frequency corresponding to $\delta = 0.5$ mm is 17.4 Khz. For a wire area $A_w = 3.142 \times 10^{-6}$ m^2, Eq. 1 gives the steady-state resistance

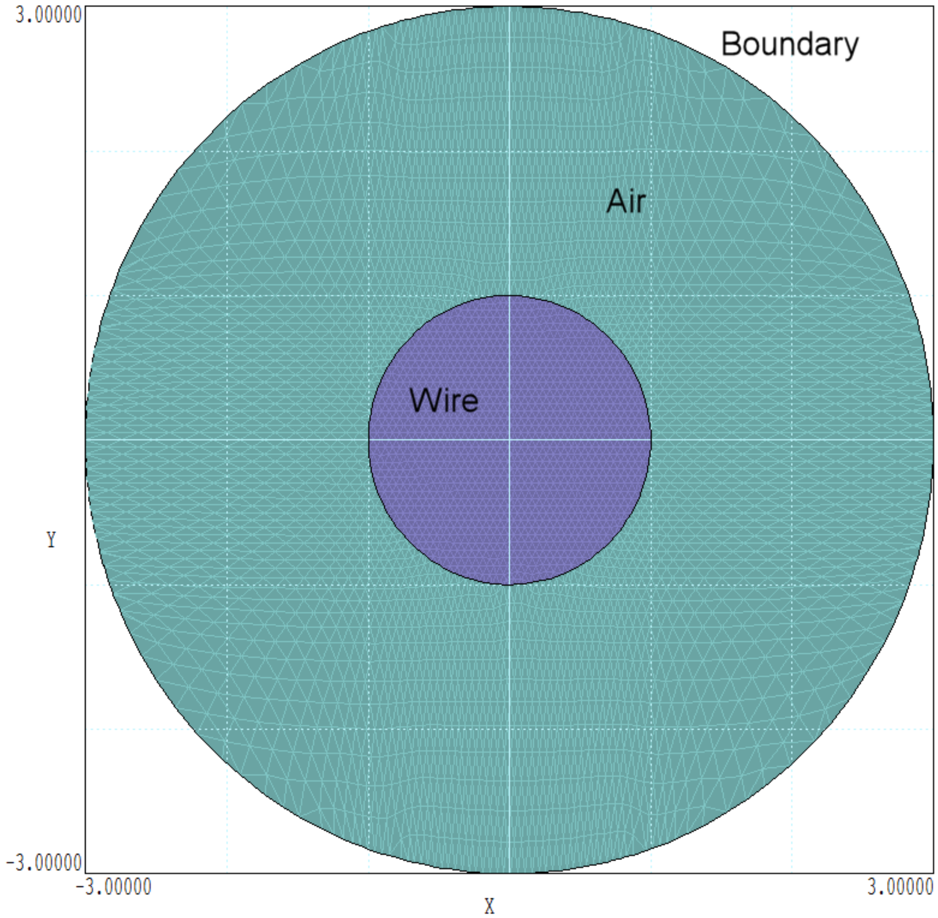

Figure 1: Mesh for the **Nelson** circular wire calculation.

$R = 5.474 \times 10^{-3}$ Ω/m. For a drive current amplitude $I_0 = 1.0$ A, the low-frequency power loss is $\overline{P} = 2.738 \times 10^{-3}$ W/m.

The program **Nelson** finds field magnetic distributions in the frequency domain – all quantities (including drive currents) vary harmonically at the same frequency. The input files to define the mesh and to control the calculation are `circular.min` and `circular.nin`. Figure 1 shows the mesh with dimensions in millimeters. The wire extends an infinite distance out of the page. The large air region was included so that the calculation could easily be modified for non-circular wires – the distant boundary has little effect on the magnetic fields inside a wire.

In the **Nelson** control script `circular.nin`, the boundary is a surface of fixed vector potential (flux conserving boundary) and the air region has $\mu_r = 1.0$ and $\sigma = 0.0$ S/m. The copper wire has $\mu_r = 1.0$, $\sigma = 5.814 \times 10^7$ S/m and a drive current $I_0 = 1.0$ A at a phase of 0.0°. The calculation of eddy currents in the wire uses the multi-stage, self-consistent method described in Chap. 4 of the **Nelson** manual. Figure 2 shows lines of **B**. At low frequency, $|\mathbf{B}|$ inside the wire increases linearly with distance from the axis, consistent with uniform current density. At high frequency, the magnetic flux density is concentrated on the surface.

To make a quantitative comparison, we can use the automatic volume integral in the **Nelson** *Analysis* menu to find the total power dissipation per length along z. Results of calculations at several frequencies are plotted in Fig. 3. The theoretical steady-state value is shown as a dashed line. As expected, power losses increase significantly in the range 10-20 kHz. At hig frequency, the current is confined to a thin layer on the wire surface. In this case. the power loss increases approximately as $\sqrt{f}$, proportional to $1/\delta$.

It is important to recognize that the numerical results for the 1.0 mm copper wire may be applied to circular wires of any diameter or composition. It is not necessary to repeat the calculation for each special case. The strategy is to recognize scaling laws and to incorporate them into a universal curve. As an example, the green line of Fig. 4 plots the power level relative to the steady-state value as a function of the ratio of the wire dimension to the skin depth. There are two advantages to applying scaling laws:

> Generality – the graphed data may used to determine the effective resistance of any circular wire.
>
> Identification of trends – it's evident thatt the power loss is proportional to $1/\delta$ at high frequency.

As an example, suppose we have a 1/8"' diameter stainless steel rod and we want to find the frequency limit such that the power loss is no more than 50% of the low-frequency value. An inspection of Fig. 4 shows that δ must be greater than $r_w/2.5$. With $\sigma = 1.240 \times 10^6$ S/m and $r_w = 1.59$ mm, the frequency corresponding to $\delta = 0.636$ mm is $f = 503.7$ kHz. To check the result, I set up another **Nelson** calculation with modified wire radius and conductivity. For $I_0 = 1.0$ A, the predicted steady-state power dissipation is 50.77 mW/m. The **Nelson** calculation gives 50.77 mW/m at $f = 1.0$ Hz and 76.28 mW/m at 503.7 kHz. The ratio of the two power levels is 1.50.

The results for circular wires could have been derived analytically with some effort. The true advantage of a numerical approach is that complex shapes are no more difficult to handle than simple shapes. To illustrate, I set up a run for square copper wires by changing the shape of the wire region in

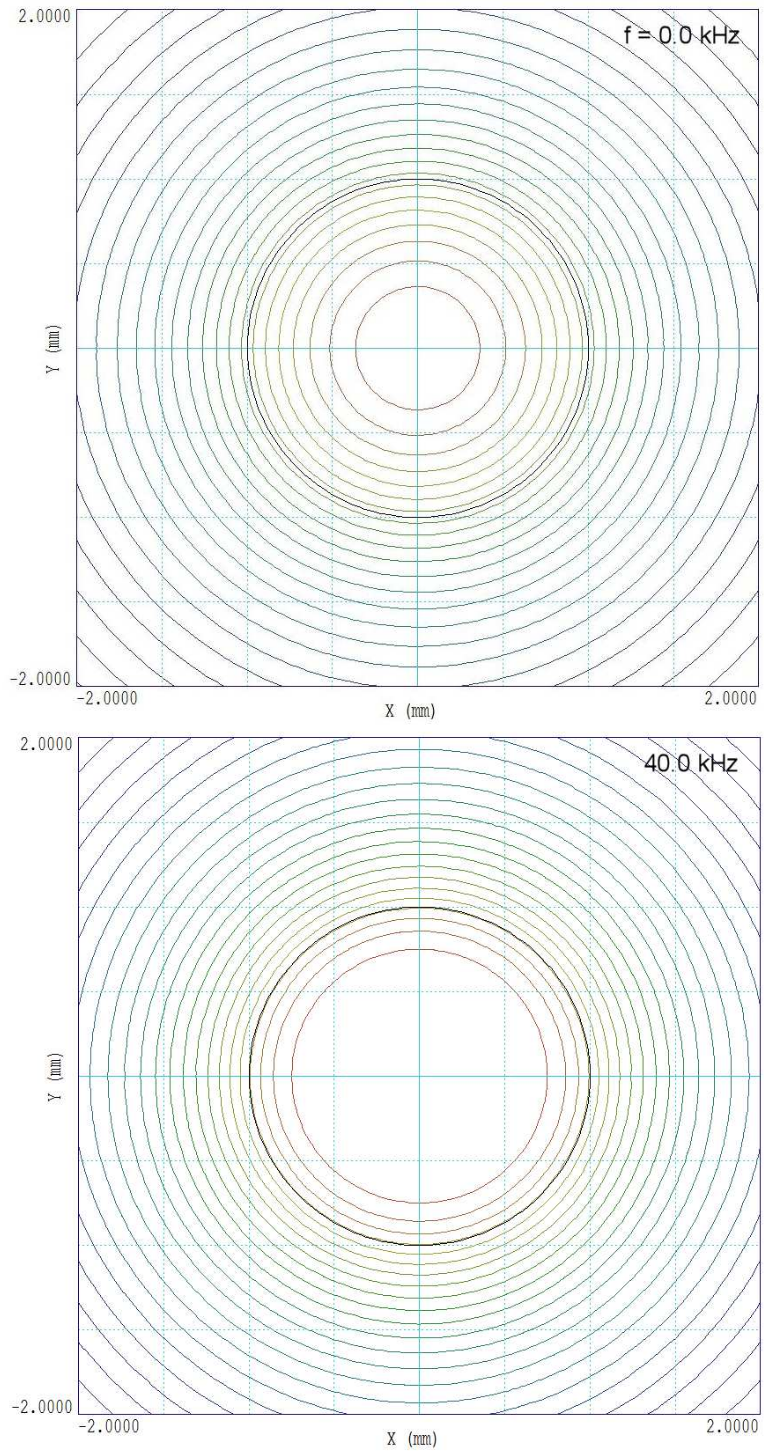

Figure 2: Lines of magnetic flux density in and around a circular copper wire at low and high frequency.

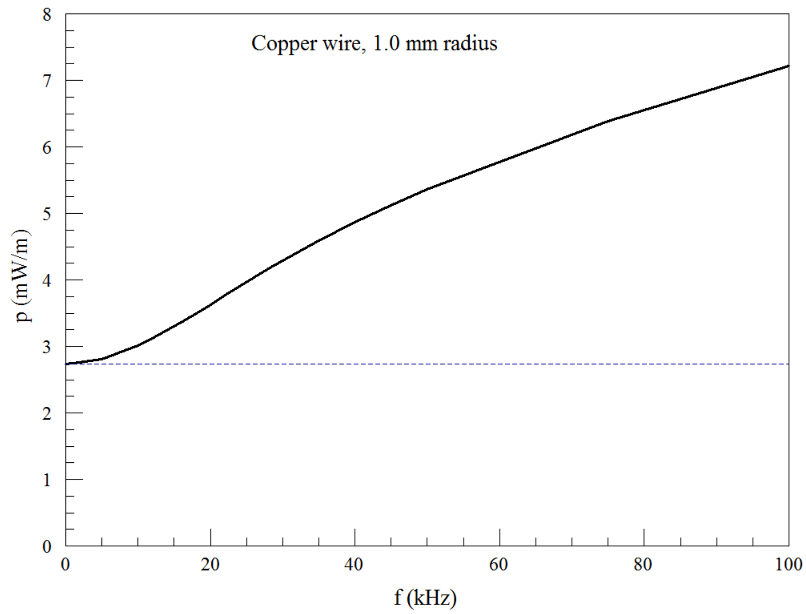

Figure 3: Power dissipation per meter in a copper wire of radius 1.0 mm as a function of frequency with a drive current amplitude of 1.0 A.

the **Mesh** input file. I used a wire with side lengths $D = 2.0$ mm. Figure 5 shows lines of $\mathbf{B}$ at high frequency. The universal power dissipation curve for square wires is plotted as the blue line in Fig. 4.

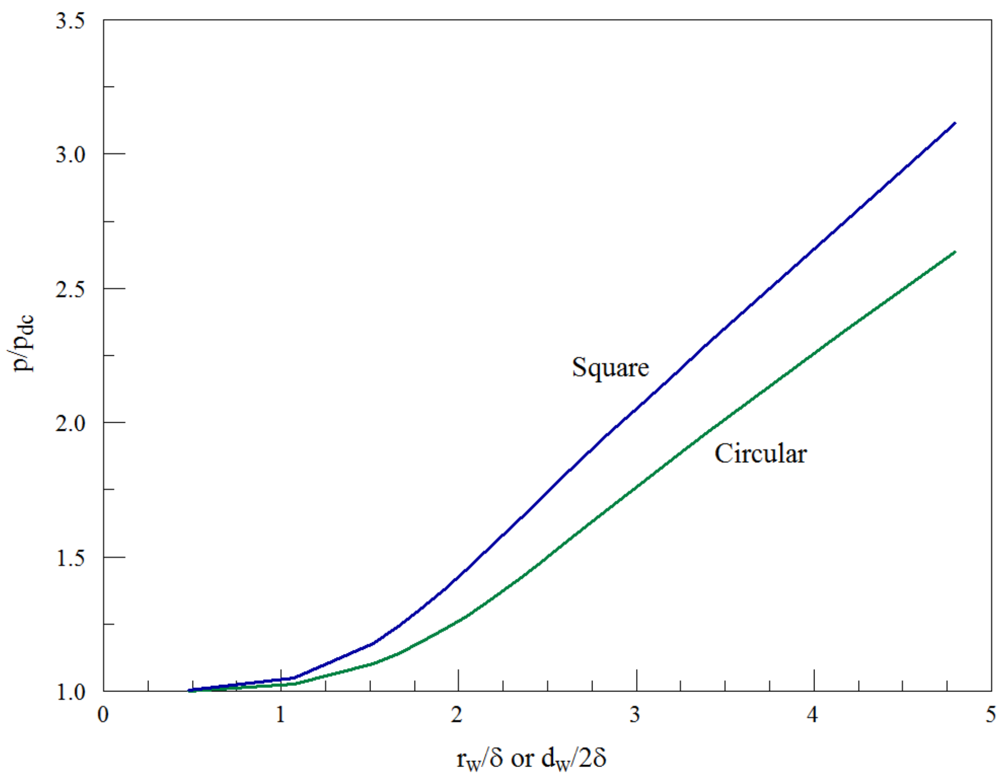

Figure 4: Universal curve for high-frequency wire resistance, power relative to the steady-state value plotted as a function of the wire dimension normalized by the skin depth. Circular and square wire cross sections.

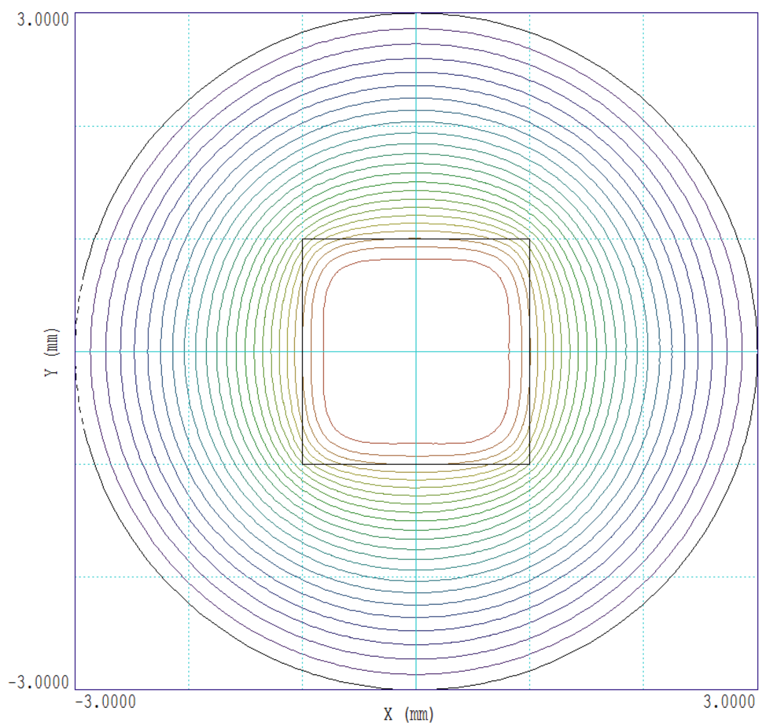

Figure 5: Lines of magnetic flux density in and around a square copper wire at 40 kHz.