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Minimizing electric fields on finite-length coaxial electrodes

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To reduce the possibility of breakdowns, a common goal in high-voltage engineering is to minimize peak electric field values on electrodes for a given voltage. One example is the choice of conductor and shield radii for a coaxial cable. For a given shield radius r_o, what choice of the conductor radius r_i gives the lowest electric? The field on the inner conductor is high when it is a thin wire ($r_i \ll r_o$) or when the its surface is close to the shield ($r_i \cong r_o$). We expect a minimum at an intermediate value. The electric field on the inner conductor of a coaxial cable with applied voltage V_0 is

$$E_r(r_i) = \frac{V_0}{r_i\ \ln(r_o/r_i)}. \tag{1}$$

Setting $\partial E_r/\partial r_i = 0.0$, we find that the minimum field occurs when

$$\frac{r_i}{r_o} = \frac{1}{e}, \tag{2}$$

where $e = 2.718$. Similarly, the condition for minimum field between spherical electrodes with radii r_i and r_o is

$$\frac{r_i}{r_o} = \frac{1}{2}. \tag{3}$$

In a recent electron-gun design project, the cathode was located on the spherical tip of a cylindrical electrode of radius r_i. For reliable gun operation, it was important to determine the peak electric field as a function of r_i for the combined geometry. The application was ideal for numerical methods – an analytic solution would be require complex series expansions and would not offer an accuracy advantage. Figure 1 shows the geometry for a calculation with the two-dimensional **EStat** code. The solution volume represented a 5.0 cm length of a coaxial transmission with outer radius $r_o = 1.0$ cm and applied voltage $V_0 = 1.0$ V. For good accuracy, the element size was about 0.01 cm. I made calculations for several choices of r_i. It was relatively easy to change the geometry by direct editing of the text-format **EStat** control script. A geometry change was effected with a global replacement of the r_i values in the vectors that defined the inner conductor.

For each solution I viewed a plot of $|\mathbf{E}|$ in the **EStat** analysis menu. In the auto-normalization mode, the code determined and displayed the maximum value of electric field magnitude in the solution volume. The color-coding in Fig. 1 shows the field distribution near the optimal value of r_i. The highest field occurred near the transition from a spherical to cylindrical surface. The circles in Fig. 2 show $|\mathbf{E}_{max}|$ as a function of the cathode radius. As expected, the best choice of r_i was somewhat below the value for ideal spherical electrodes (Eq. 3).

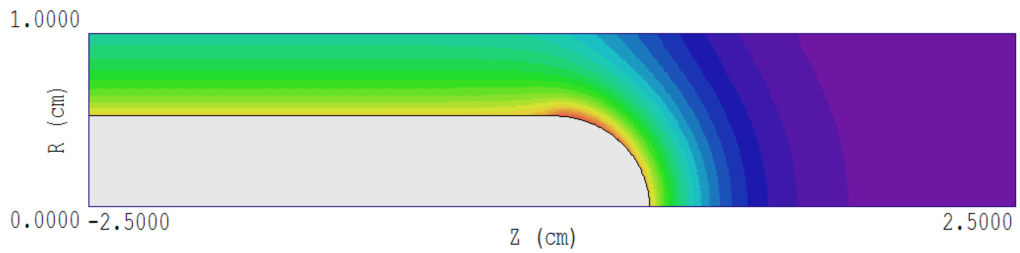

Figure 1: Calculation geometry, coaxial transmission line with $r_o = 1.0$ cm with a center conductor of radius r_i with a spherical tip. The color coding shows for electric field distribution for $r_i = 0.5$ cm

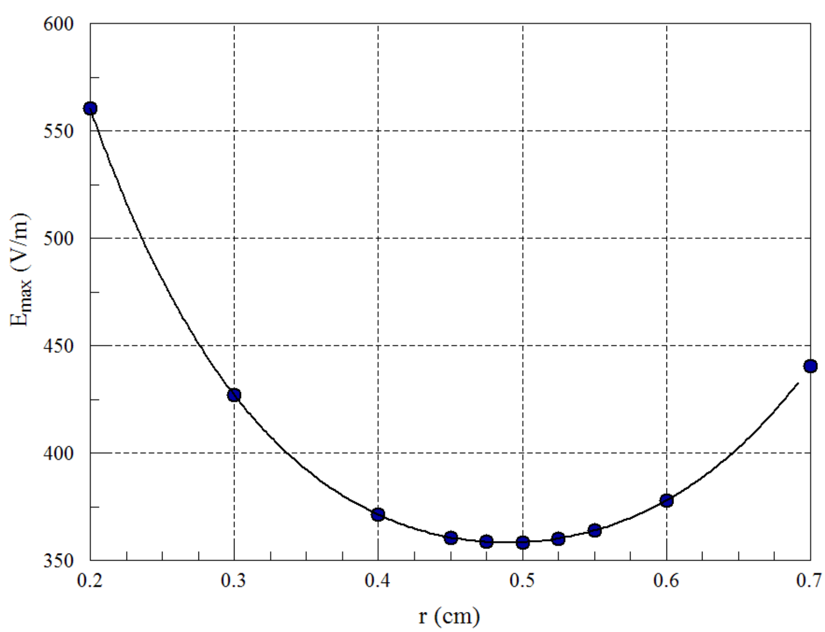

Figure 2: Peak electric field magnitude as a function of the cathode radius.

To find an accurate value of r_i with a finite number of calculations, I made a fifth order polynomial fit to the data points (solid line in Fig. 2). This task took only a few minutes using the numerical utility program PsiPlot (http://www.polysoftware.com/plot.htm). Rather than attempt to solve the resulting equation, I simply instructed the program to calculate a large number of instances over a short interval near the minimum. The best choice of inner radius was

$$\frac{r_i}{r_o} = 0.490. \tag{4}$$

At this value, the peak electric field for the normalized calculation was 358.2096 V/m or $3.5821(V_0/r_o)$.