



Two-dimensional space-charge-limited charged particle flow

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The maximum current density of non-relativistic charged particle flow across an infinite planar gap is described by the familiar Child law:

$$j_c = \left(\frac{4\epsilon_o}{9}\right) \sqrt{\frac{2q}{m}} \frac{V_o^{3/2}}{D^2}. \quad (1)$$

In the equation, V_0 is the applied voltage, m is the particle mass, q is the charge and D is the gap spacing (MKS units).

The 2D calculations of this report describe space-charge limited flow across a planar gap from finite emission areas (circles and slots). Under the assumption of uniform emission current density, the following analytic result was derived for a circular source by Lau¹:

$$\frac{I}{I_c} = 1 + \frac{D}{4R}. \quad (2)$$

Here, I is the total emission current, R is the source radius and

$$I_c = j_c (\pi R^2). \quad (3)$$

The result was confirmed with particle-in-cell simulations by Luginsland, et.al.² under the same limiting assumptions in the range $D/R > 2$. The quantities in Eq. 2 represent an effective choice to display the results. Even though the current I may vary with R over many orders of magnitude, the quantity I/I_c remains close to unity. It represents the enhancement of flow in a two-dimensional geometry where the beam spreads over an area larger than the emission surface, reducing virtual cathode effects. Equation 2 also suggests the use of (R/D) as a scaling parameter in numerical calculations. If we perform calculations for a specific choice of Z , m , V_0 and D with variations of R , then the results can be applied to systems of any non-relativistic particles and any geometry by applying the scaling laws. The two-dimensional calculations have practical as well as academic significance. For example, we can predict the maximum possible extracted electron current from a laser spot on a planar photo-emitter.

This report describes numerically-exact results generated with the **Trak** beam program³. The code uses the ray-tracing technique⁴ for high-accuracy simulations of steady-state, self-consistent charged particle flows. The new results extend the previous work in three areas:

¹Y. Y. Lau, Phys. Rev. Lett. **87**, 278301 (2001).

²J. W. Luginsland, Y. Y. Lau, R. J. Umstadtd, and J. J. Watrous, Phys. Plasmas **9**, 2371 (2002).

³S. Humphries, J. Comp. Phys **125**, 488 (1996)

⁴W. B. Herrmannsfeldt, Stanford Linear Acc. Center, **SLAC-331**, 1988 and A. C. Paul, Lawrence Berkeley Lab, **LBL-13241**, 1982.

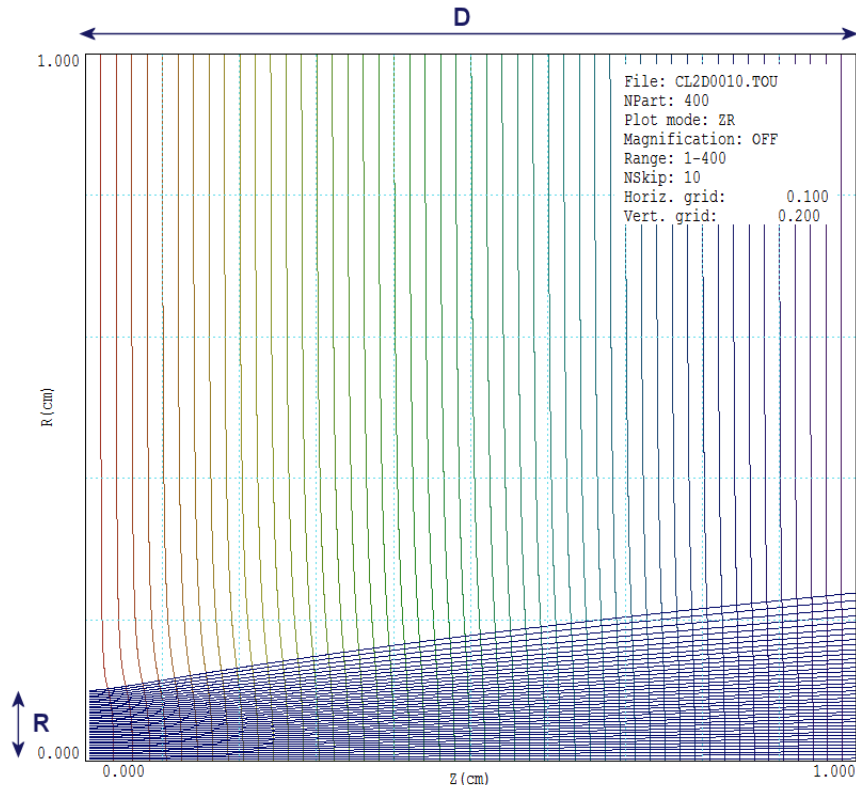


Figure 1: Calculated orbits and equipotential lines for $R = 0.1D$. One tenth of the orbits are plotted

- The condition of uniform emission is removed giving a self-consistent determination of current density variations over the source.
- The method applies in the range $D/R \gg 1$ (*i.e.*, small radius sources).
- The calculations give detailed information about the extracted beam quality.

The **Trak** calculations were performed for non-relativistic electrons with $D = 1.0$ cm and $V_0 = 5000$ V. We checked eight source radius values over the range $R = 5D$ to $R = 0.025D$. It was necessary to prepare optimized meshes for each value to maintain the number of ray traces at around 400 and to maintain an emission gap spacing small compared to R . Figure 1 shows the geometric parameters and calculated results for $R = 0.1D$. For clarity, only 10% of the traces are plotted. The curvature of the equipotential lines shows the effect of the beam space charge. Figure 2 shows the main result, the enhancement of extracted current over the simple application of the planar law of Eq. 3. The plot shows $(I/I_c - 1)$ determined by **Trak** along

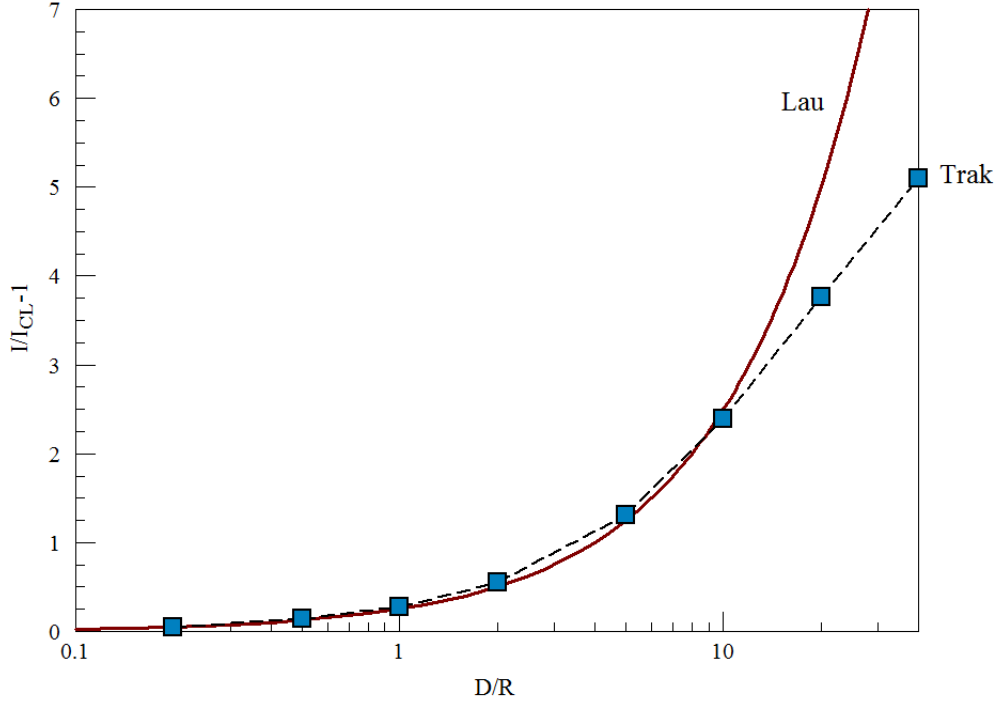


Figure 2: **Trak** calculation of $(I/I_c - 1)$ as a function of (D/R) for a source of radius R and an acceleration gap of width D . The solid line shows the prediction of Eq. 2.

with the Lau prediction, $(D/4R)$. Agreement is good for larger source radii, $R > 0.1D$ but deviates for small radius sources.

The difference probably results from the assumption of uniform emission current density in the analytic and particle-in-cell models. Figure 3 plots the current density at the emission surface determined by **Trak**. When the source radius is much larger than the gap width (Fig. 3a), the current density is close to the value 0.8238 A/cm^2 predicted by Eq. 1 over most of the source with enhancement at the edge. In contrast, at small radius (Fig. 3b), the current density exceeds the one-dimensional Child law prediction with considerable variation over the full source area.

The source radius also affects the quality of the beam. Figure 4 plots r' of trajectory as a function of r for large (Fig. 4a) and small (Fig. 4b) radius sources. Because of the non-linear transverse forces, the large radius beam has significant effective emittance, while the small source distribution follows almost a straight line. Hence, high perveance sources require focusing electrodes for a low-emittance beam. Although the small source gives better beam quality, keep in mind that the extracted current is much smaller (30.9 mA versus 3.30 A).

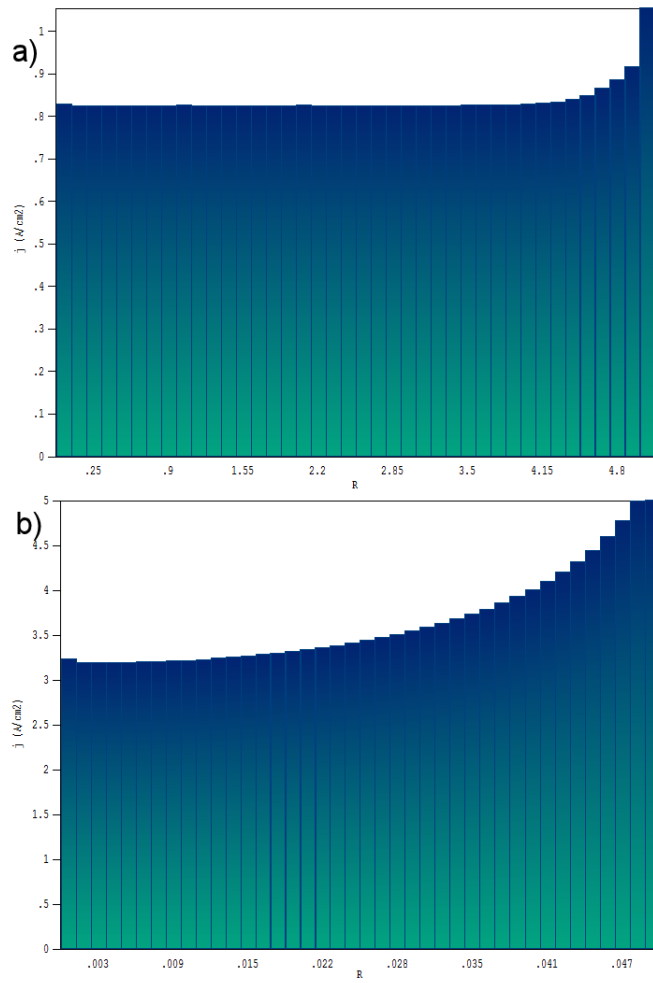


Figure 3: Emission current density as a function of radius for *a*) $R/D = 5.0$ and *b*) $R/D = 0.05$.

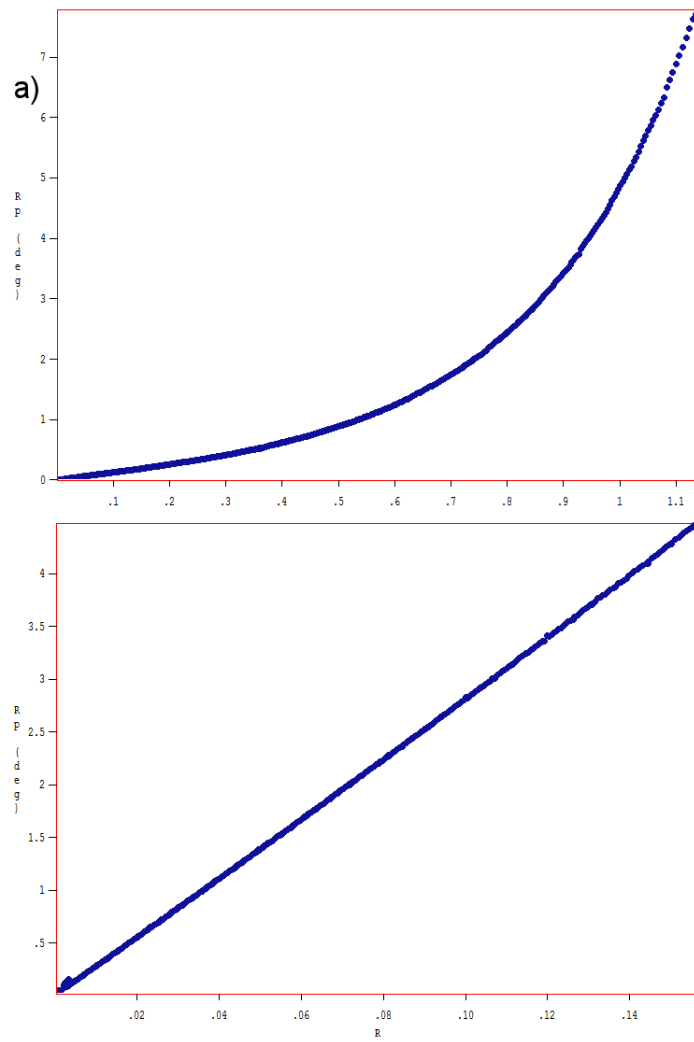


Figure 4: Radial phase space distribution (r' versus r) at the exit plane ($z = 1.0$ cm) for a) $R/D = 1.0$ and b) $R/D = 0.05$.

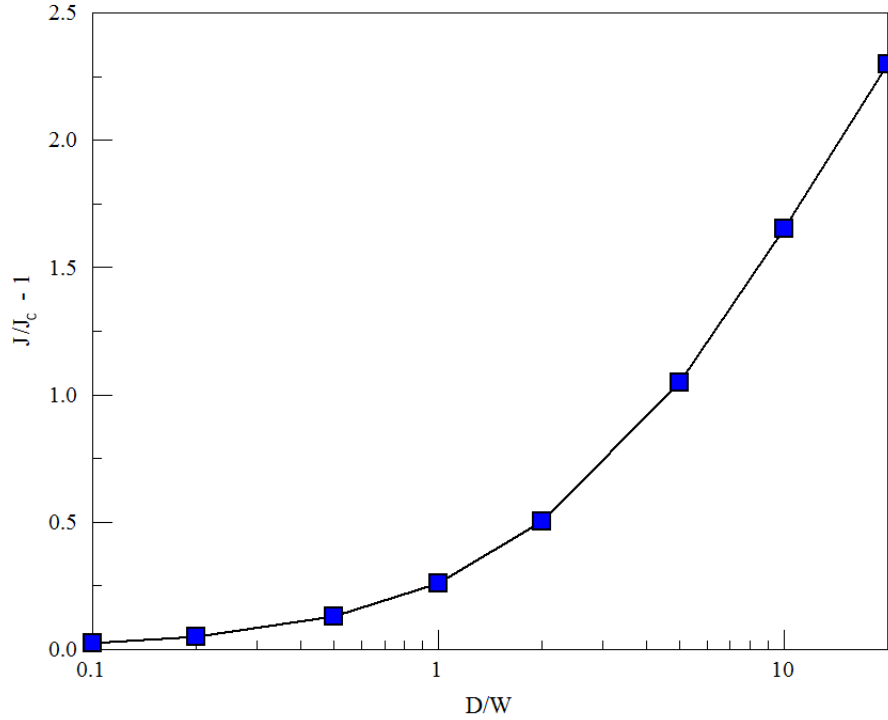


Figure 5: The quantity $(J/J_c - 1)$ as a function of (D/W) for a long slot of full-width W and an acceleration gap of width D . The quantity J is the linear current density current per length along the slot and $J_c = wj_c$.

Finally for completeness, Figure 5 shows the linear current enhancement calculated by **Trak** for a planar two-dimensional geometry. The quantity W is the full width of an emission slot of infinite length and again D is the width of the planar acceleration gap. Here, J is the linear current density (current per length along the slot) and $J_c = wj_c$.

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