Modeling radioactive sources with Monte Carlo codes

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Processes represented by **GamBet** often involve interactions with atomic electrons (*e.g.* bremsstrahlung emission of photons and Compton electrons). Particle emission may also follow from reactions within nucleii. Although **GamBet** does not address nuclear physics, the program can determine the histories of particles created by nuclear events. There are two types of processes: *nuclear reactions* and *radioactivity*. A nuclear reaction is a response to a specific event, like fission following absorption of a neutron. On the other hand, radioactive events occur spontaneously at random times. Some nucleii are inherently unstable but in a state of local equilibrium. A transition can occur through quantum tunneling, a random process. For X-ray applications, we will limit attention to radioactivity. To begin, we'll review some nomenclature and characteristics of sources. The second part of the report deals with specific **GamBet** strategies.

Four types of particles may be generated in radioactive events:

Fast electrons and positrons (beta rays)

Photons (gamma rays)

Heavy charged particles (protons, alpha rays,...)

Neutrons

This report deals with first two types. Radioactive sources of β and γ rays have extensive applications in areas such as medical treatments, food irradiation and detector calibration.

The activity of a radioactive source is determined by law of radioactive decay:

$$\frac{dN}{dt} = -\lambda N. \tag{1}$$

The quantity N equals the total number of nucleii in the source. The left-hand side is the number of nucleii that decay per second. The quantity λ (with units of s^{-1}) is the *decay constant*. It depends on the energy state and quantum barrier of the nucleus. Accordingly, sources exhibit huge variations of λ. The historical unit of activity for a source is the curie (Ci). One curie equals 3.7×10^{10} decays/s (approximately equal the activity of 1 gram of Ra^{226}). The modern standard unit is the becquerel (Bq) equal to 1 decay/s ($1\ \text{Bq} = 2.703 \times 10^{-11}$ Ci).

We can also interpret the decay constant in terms of a single nucleus. The probability that a nucleus has not decayed after a time t is

$$p(t) = \exp(-\lambda t). \tag{2}$$

The *average lifetime* (the mean of the distribution) is $1/\lambda$. The *halflife* is another useful quantity. It equals the time for half of the nucleii present in a source at $t = 0.0$ to decay. Equation 2 implies that

$$t_{1/2} = \frac{\ln(2)}{\lambda}. \tag{3}$$

In comparison to particle accelerators, the main advantage of radioactive isotopes as sources is that they do not require power input and expensive ancillary equipment (*e.g.*, power supplies, vacuum systems,...). Many isotopes are produced by exposure in a nuclear reactor and may be relatively inexpensive when reactors are available. The disadvantages of radioactive sources are that they run continuously and produce a broad energy spectrum of electrons and positrons.

The most important nuclear processes for the production of beta and gamma rays are *beta decay* and *electron capture*. Figure 1 shows the atomic mass of the most stable isotopes as a function of atomic number Z. Isotopes above the line have an excess of neutrons – their usual route toward stability is to emit a β^- particle (electron), converting a neutron to a proton while preserving the number of nucleons. In other words, the nucleus changes its isotopic identity while preserving its isomer identity. Similarly, isotopes with an excess of protons emit β^+ particles (positrons). Both forms of nuclear transformations are called *beta decay*.

First, consider β^- emission. There are two isotopes commonly used in research and industry: Cs^{137} and Co^{60}. Nuclear processes are commonly illustrated with energy-level diagrams – Fig. 2*a* shows the decay scheme for Cs^{137}. The horizontal axis represents isomer identity and the vertical axis shows energy levels. Dark lines indicate a nucleus in the ground state and light lines designate an excited state. The arrows indicate the directions of transformations. The starting point is the ground state of Cs^{137}. The value 30.17 years is the half-life for decay. A decay event of type β^- converts the nucleus to the more stable isomer, Ba^{137}. The arrows indicate that there are two decay paths. In 94.6% of the decays, the emission process carries off 0.512 MeV (shared between the emitted electron and an anti-neutrino) and leaves the Ba^{137} nucleus in an excited state. The state decays with a half-life of 2.55 minutes, resulting in emission of a 0.662 MeV gamma ray. In 5.4% of the events, the β^- particle and antineutrino carry of 1.174 MeV and leave the product nucleus in the ground state.

The emission process does not produce a single β^- particle of energy 0.512 or 1.174 MeV, but rather a broad spectrum of electrons with kinetic energy spread between zero and the maximum. The reason is the requirement for conservation of spin. Nucleii have spin values an integer multiple of $h/2\pi$ while electrons have spin $\frac{1}{2}$ $(h/2\pi)$. For balance, an additional particle

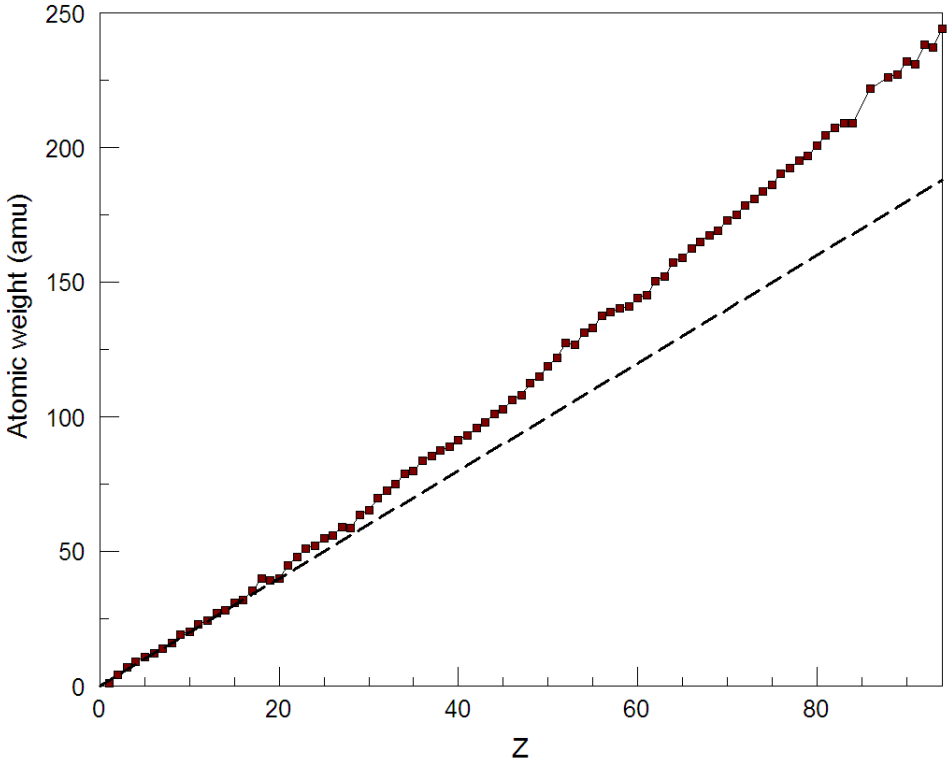

Figure 1: Atomic weight of the most stable isotopes as a function of atomic number Z. The dashed line shows the mass for equal numbers of neutrons and protons.

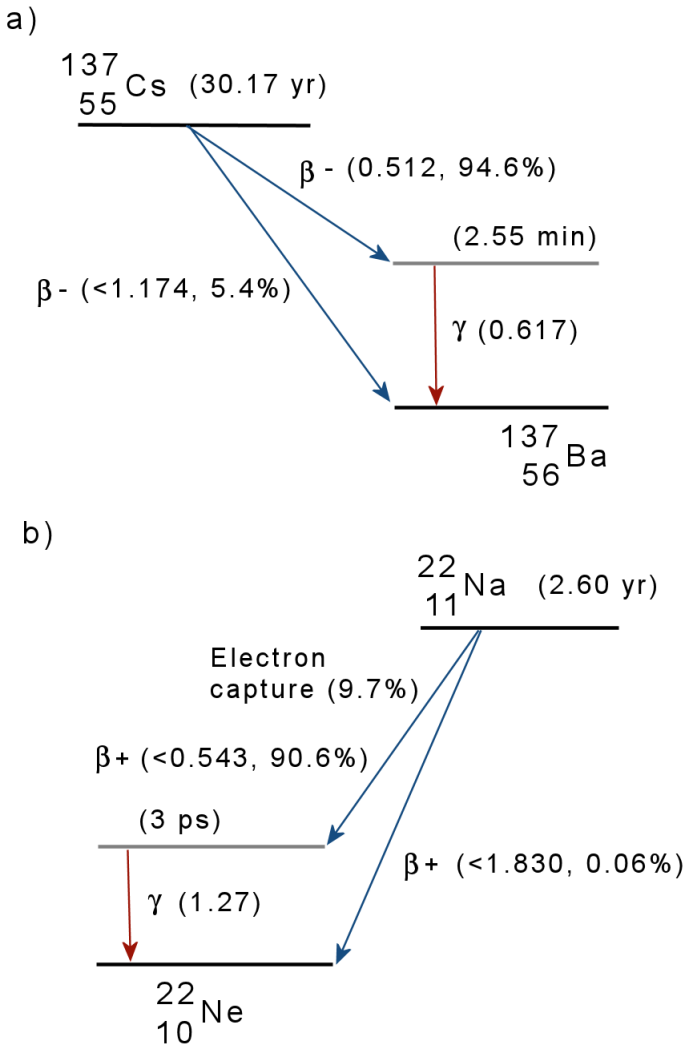

Figure 2: Energy level diagrams for the radioactive decay of Cs^{137} and Na^{22}.

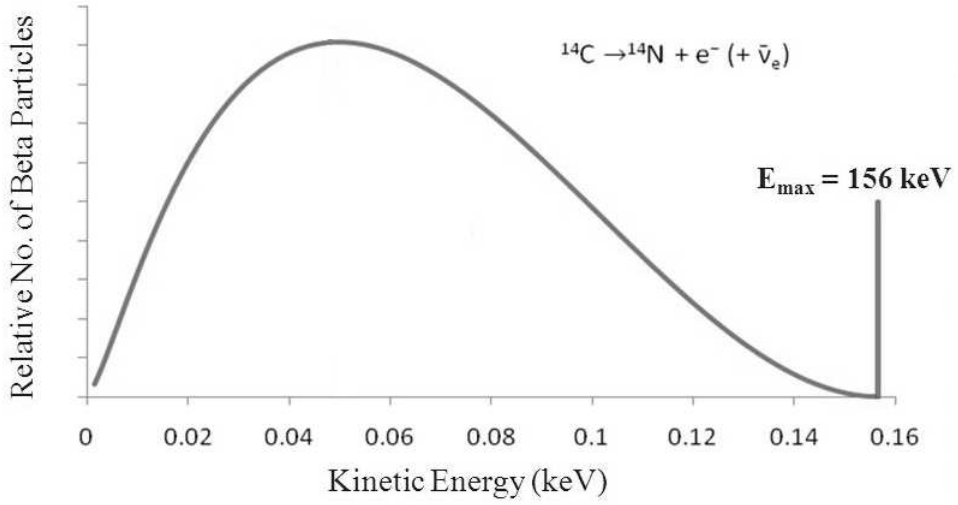

Figure 3: Distribution of β^- energy for the radioactive decay of C^{14}.

is required with half-integer spin. In his theory of beta decay, Fermi postulated the existence of neutrinos and antineutrinos, neutral particles with spin $\frac{1}{2}$ $(h/2\pi)$ and very small mass, thereby almost undetectable. In a β^- decay, the available energy is partitioned between the electron, the nucleus and an antineutrino. The theory to determine the spectrum is complex – all β^- decays give rise to a spectrum similar to that of Fig. 3. The spectrum is skewed toward lower energy by the effect of Coulomb attraction as the electron escapes from the nucleus. Generally, Cs^{137} is used as a a source of 0.662 gamma rays because the β^- particles are preferentially absorbed by the source and surrounding structure and the antineutrinos pass away with no effect.

We next consider proton-rich isotopes that approach the stability line through emission of positrons. The mechanism is similar to β^- emission with the exceptions that a neutrino is emitted and the positron spectrum is shifted toward higher energies because of Coulomb force repulsion from the nucleus. Figure 2*b* shows the energy-level diagram for Na^{22}, a positron emitter. The halflife for all decay processes is 2.60 years. There are several decay pathways. The most likely event (90.33% probability) is that a proton changes to a neutron by emission of a positron, leaving the product isotope Ne^{22} in an excited state. A gamma ray of energy 1.275 MeV is released almost immediately as the nucleus relaxes to the ground state. In this case, the maximum positron energy is 0.545 MeV. In rare instances, a positron with energy $\leq$ 1.82 MeV is released, leaving the product nucleus in the ground state. A third process that may occur is electron capture. In 9.62% of the decays, an inner orbital electron is captured by the nucleus, again resulting

Name	Gamma energies	Beta energies	Half life
Barium-133	0.081, 0.276, 0.303, 0.356, 0.384		10.7 years
Cesium-137	0.032, 0.662	0.512, 1.173	30.1 years
Cadmium-109	0.088		465 days
Cobalt-57	0.122, 0.136	0.019^+	271 days
Cobalt-60	1.173, 1.333	0.318	5.27 years
Manganese-54	0.835	0.542	312 days
Sodium-22	0.511, 1.275	0.546^+	2.6 years
Tin-113		0.014	115 days
Strontium-90		0.546	28.5 years
Thallium-204		0.764	3.78 years
Zinc-65	1.116	0.236, 1.352	244 days

Figure 4: Common commercial radioactive sources

in the conversion of a proton to a neutron. The Ne^{22} nucleus is left in the same excited state as with β^+ emission, again followed by the release of a 1.27 MeV γ. The difference from β^+ decay is that no positron or neutrino is emitted. Electron capture leaves a vacancy in the K or L shell of the electron cloud, so characteristic X-rays are also emitted as the atom relaxes.

Figure 4 contains a list of useful commercial radioactive sources of electrons, photons and positrons. A common feature is a halflife of one to a few years. For isotopes with lower values, it would be necessary to produce and use them quickly. A long half life means reduced activity.

We'll now turn to **GamBet** modeling techniques, in particular how to create a particle input file to represent a radioactive source. There are some challenges:

> Particles are emitted over an extended spatial region, the volume of the source.
>
> Electrons and positrons have a broad energy distributions.
>
> Often, we want to normalize particle flux to represent a specific source activity.

Particle file creation is greatly facilitated through the use of statistical codes like **R**[1]

Dealing with the finite source size is relatively easy. If the activity is uniform over the source volume, then the probability density for emission is uniform over the volume. As an example, consider a cylindrical source of length L and radius R. Given a routine that creates a random variable ξ in

[1]A comprehensive short course on using **R** with **GamBet** can be found at the following Internet site: `http://www.fieldp.com/rintroduction.html`.

the range $0 \leq \xi \leq 1.0$, then values of the z coordinate (along the cylinder axis) are assigned according to

$$z = L(\xi_1 - 0.5). \tag{4}$$

We can use the rejection method to determine coordinates in the x-y plane. We assign coordinates by

$$x = 2R(\xi_2 - 0.5), \tag{5}$$
$$y = 2R(\xi_3 - 0.5). \tag{6}$$

and keep only instances where

$$x^2 + y^2 \leq R. \tag{7}$$

With regard to energy distributions, photons from sources like Cs^{137} and Na^{22} are essentially monoenergetic. In contrast, the β particles have an energy distribution like that of Fig. 3. In principle, thin films could be used as sources of electrons or positrons. In this case, it would be necessary to represent the spectrum and to determine the effect of energy loss in the film The spectral shape and endpoint energy vary with the type of isotope. Chapter 10 of the reference **Using R for GamBet Statistical Analysis** (`www.fieldp.com/rintroduction.html`) discusses methods for creating arbitrary distributions. In practice, an exact model may not be necessary and the data may not even be available. In applications such as estimating shield effectiveness, it may be sufficient to model the β decay spectrum with a simple function like

$$p(E)dE = \frac{\pi}{2E_{max}} \sin\left(\frac{\pi E}{E_{max}}\right), \tag{8}$$

where E_{max} maximum β energy. Taking the integral gives the cumulative probability distribution (i.e., the probability that a β has energy less than or equal to E):

$$P(E)dE = \frac{1 - \cos(\pi E/E_{max})}{2}, \tag{9}$$

where values of $P(E)$ range from 0.0 to 1.0. We can obtain the desired distribution by assigning energy from a random-uniform variable ξ using the inverse of Eq. 9

$$E_i = \frac{E_{max}}{\pi} \cos^{-1}(1 - 2\xi_i). \tag{10}$$

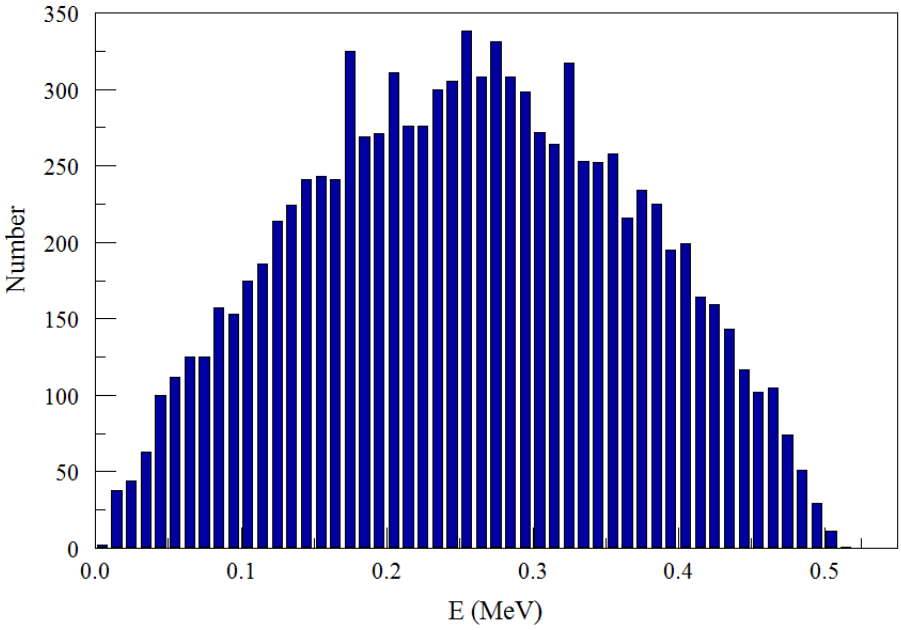

Figure 5: Generation of the spectrum of Eq. 8 with 10,000 particles using assignment from Eq. 10

Figure 5 shows the application of Eq. 10 with 10,000 particles having endpoint energy $E_{max} = 0.512$ MeV.

To conclude, we'll address how to create a **GamBet** source file to represent a given source activity. We'll follow a specific example – a Co^{60} source with activity 10 Ci. This figure corresponds to a disintegration rate of $R_d = 3.7 \times 10^{11}$ s^{-1}. Figure 6 shows an energy level diagram. The isotope (produced in a reactor) decays through β^- decay with a halflife of 5.27 years. Almost all events result in a excited state of the Ni^{60} nucleus that relaxes to the stable ground state by rapid emission of γ rays of energy 1.17 and 1.33 MeV. A source assembly typically consists of the source combined with shielding and collimators to create a directional photon flux. A goal of a calculation could be to compare radiation fluxes in the forward and reverse directions.

We specify $N_p = 1000$ model emission points uniformly distributed over the source volume using techniques like those discussed previously. At each emission point, we generate $N_g = 500$ photons of energy 1.17 MeV and N_g photons of energy 1.33 MeV. The photons are randomly distributed over 4π steradians of solid angle. The following equations can be used to pick the azimuthal and polar angles:

$$\phi = 2\pi\xi_1, \tag{11}$$

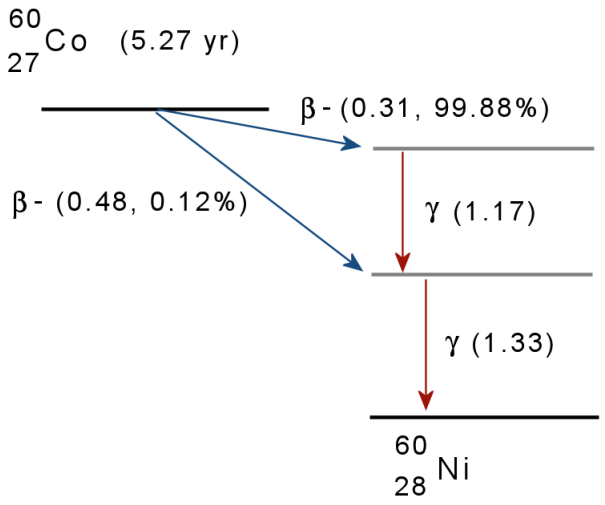

Figure 6: Energy level diagram for the radioactive decay of $C0^{60}$.

$$\theta = \sin^{-1}(2\xi_2 - 1). \tag{12}$$

In the continuous-beam mode of **GamBet**, each photon in the file should be assigned the flux

$$F = \frac{R_d}{N_p N_g} = 7.40 \times 10^5 \quad (\text{s})^{-1} \tag{13}$$

In this case, **GamBet** gives absolute values of particle flux through and deposited dose in structures surrounding the source assembly. Note that this example is relatively simple because almost all events follow the same decay path. In the case of Cs^{137} (Fig. 2), we need to multiply R_d by 0.946 to get the correct absolute flux of 0.617 MeV γ rays.

The procedure as described may be inefficient to calculate forward photon flux or shielding leakage because most of the model particles would not contribute. A simple variance reduction technique is to limit the range of solid angle $d\Omega$ so that photons are preferentially directed toward the measurement point. The solid angle should be large enough to include the possibility of scattering from the shield or collimator. To properly normalize the calculation, the photon flux values should be adjusted by a factor $d\Omega/4\pi$.