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Magnetic shield design with numerical methods

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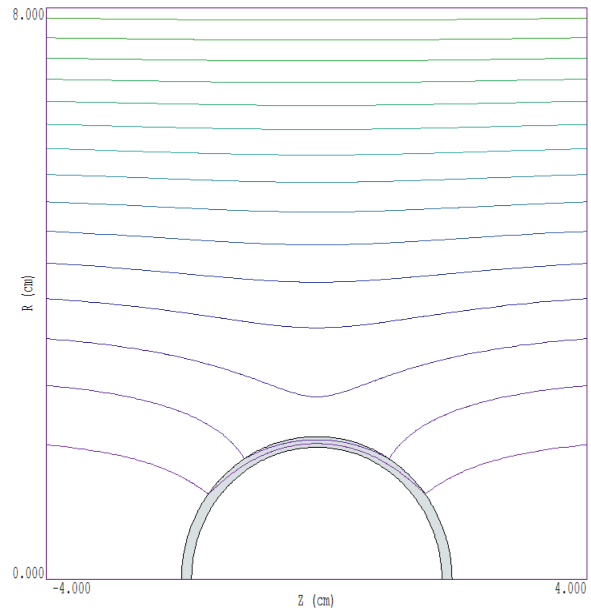

Figure 1: Hollow steel sphere with $\mu_r = 833.3$ immersed in a uniform magnetic field, showing lines of magnetic flux density $\mathbf{B}$. $R_0 = 2.0$ cm, $\Delta r = 0.15$ cm.

1 Ideal material response

Magnetic shielding has long been a critical concern for sensitive electro-optical devices like photomultipliers. Shielding calculations have become increasingly important for large magnets in MRI facilities. Here, shields are important for personnel safety, operation of nearby equipment and isolation of multiple MRI machines. A magnetic shield is relatively simple. It is an enclosure constructed from steel or other materials with a high relative magnetic permeability. Because lines of magnetic flux density flow preferentially through such materials, they are shunted around a protected volumes. Numerical codes such as **PerMag** and **Magnum** are effective tools for shielding design. This article reviews some useful techniques and good practices.

We shall begin by reviewing some theoretical constraints on shields with non-saturable materials characterized by a fixed value of relative magnetic permeability μ_r. Figure 1 shows an example, a hollow steel sphere immersed in a uniform magnetic field. (Note that the figure is a z-r plot and represents the full sphere.) The shell has outer radius R_o and inner radius R_i. The flux density far from the origin approaches the value $\mathbf{B} = B_o\ \mathbf{z}$. The analytic result[1] in the limit $\mu_r \gg 1$ is that the field inside the sphere has the uniform value $\mathbf{B} = B_i\ \mathbf{z}$, where

$$\frac{B_i}{B_o} = \left(\frac{9}{2\mu_r}\right)\ \frac{1}{1 - r_i^3/r_o^3}. \tag{1}$$

If the thickness of the spherical shell, $\Delta R = R_o - R_i$, is small compared to the average radius, Eq. 1 may be written as:

$$\frac{B_i}{B_o} \cong \frac{3R_o}{2\mu_r \Delta R}. \tag{2}$$

The **PerMag** solution `SHIELDSCALING` (supplied in the example library) is a numerical example of a spherical shield. The file `SHIELDSCALING.MIN` represents a shell with outer radius $R_o = 2.0$ cm inside a large solution volume (a cylinder of radius 10.0 cm and length 20.0 cm). The solution boundaries are set to generate a uniform field $B_z = 0.050$ tesla in the absence of the sphere:

> The left and right boundaries in z have the special Neumann condition so that $\mathbf{B}$ is normal to the surfaces.
>
> The stream function on the inner radial boundary has the Dirichlet condition $rA_\theta = 0.0$ tesla-m^2.
>
> The solution volume is filled with a uniform flux density $\mathbf{B} = B_o\mathbf{z}$ if the outer radial boundary has the stream function value $R_oA_\theta(R_o) = B_oR_o^2/2$. For the example, $R_oA_\theta(R_o) = 2.5 \times 10^{-4}$ tesla-m^2.

We can confirm that the assigned value of stream function is correct by making an initial run with the iron set to $\mu_r = 1.0$. An initial calculation with a thin shell ($\Delta R = 0.05$ cm) employs a small local element size of 0.025 cm. The code prediction for the internal field is $B_i = 1.187 \times 10^{-3}$ tesla, close to the theoretical prediction of $B_i = 1.200 \times 10^{-3}$ tesla.

[1] J.D. Jackson, **Classical Electrodynamics, Second Edition** (Wiley, New York, 1975), Sect. 5.12.

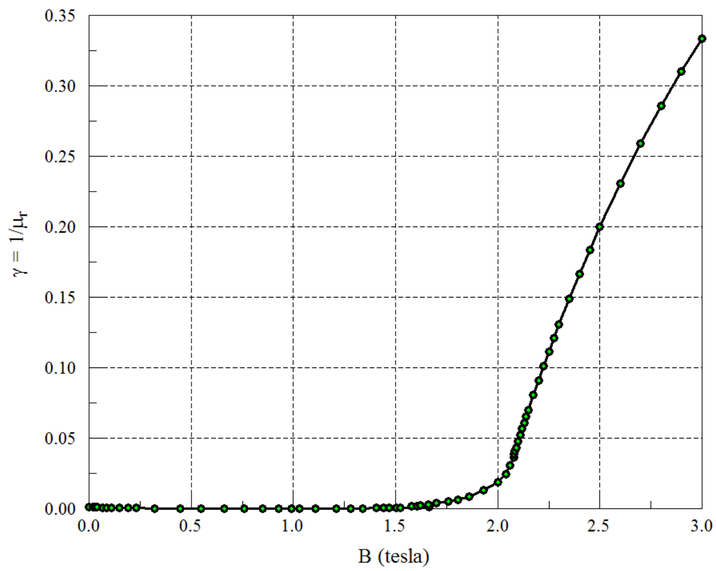

Figure 2: Saturation curve for type 50H450 steel, plot of $1/\mu_r$ versus the total flux density in the material.

2 Scaling in shielding calculations

Thin sheets are the bête noire of finite-element calculations. Large disparities in scale often lead to bad meshes and inaccurate answers. The issue may arise for magnetic shields constructed of thin layers of steel or μ-metal. In this case, it is often useful to employ scaling principles rather than a literal approach. To define an alternative, note that Eq. 2 involves the product $(\mu_r \Delta R)$. Suppose that we double the thickness of the shell and halve the magnetic permeability. The result should be about the same as long as $\Delta R \ll R_o$. In the test example of Fig. 1, if we use a thickness $\Delta R = 0.10$ cm and $\mu_r = 1250$, the calculated field inside the shield is $B_i = 1.218 \times 10^{-3}$ tesla. A thickness $\Delta R = 0.15$ cm with $\mu_r = 833.3$ gives $B_i = 1.2492 \times 10^{-3}$ tesla, about a 4% difference from the theoretical result.

As a general rule, in a two- or three-dimensional shielding calculation, you can substitute a thicker sheet with reduced μ_r as long as the actual and adjusted sheet thicknesses are small compared to the scale size of the volume being shielded. You should be cautious applying scaling if parts of the shield may become saturated. This topic is discussed in Sect. 4.

3 Material saturation

Shielding design is more complex if portions of the ferromagnetic material become saturated. In this state, all magnetic domains of the material are aligned, and it looses its ability to conduct additional flux. As a result, the shield becomes less effective. Figure 2 shows a saturation curve for a typical magnet steel. The quantity $\gamma = 1/\mu_r$ is plotted versus the magnitude of the total flux density in the material ($|\mathbf{B}|$). It is convenient to use the quantity γ because it lies in the range 0.0 to 1.0 while the relative magnetic permeability may vary by several orders of magnitude. From the discussion of Sect. 1, the reduction factor of a magnetic shield is roughly proportional to γ. The quantity is small at low field so that shielding is effective. In the plot, γ starts to increase at about 1.6 tesla and rises sharply above the material saturation point ($B_s = 2.00$ tesla).

We can estimate conditions for saturation in a shield with thickness ΔR and radius R_o normal to an applied field B_o. Inspection of Fig. 1 shows that the shield pulls in both interior and exterior flux, roughly over a radial range $0 \leq r \leq 2R_o$. The cross-section area is about $4\pi R_0^2$ while the area of the shield is $2\pi R_0 \Delta R$. Assuming all diverted flux passes through the shield material, we can find a criterion for the onset of saturation:

$$\Delta R \cong 2R_0 \left(\frac{B_o}{B_s} \right). \tag{3}$$

For example, the thickness of a shield composed of a material with $B_s = 2.00$ tesla with radius $R_o = 100.0$ mm immersed in a field $B_o = 0.05$ tesla should satisfy the condition $\Delta R > 5.0$ mm.

The **PerMag** example `SHIELDPIPE` gives a quantitative picture of the effects of saturation. It treats the practical case of an open shield, a steel pipe with outer radius $R_o = 100.0$ mm and length $L = 300.0$ mm. Figure 3 shows the geometry. The material follows the saturation curve of Fig. 2. Again, a uniform applied field $B_o = 0.05$ tesla is created by setting a fixed stream function of the outer radius of the solution volume. A sequence of runs with wall thickness varying from 2.0 to 10.0 mm were created. The results are plotted in Fig. 4. The illustration shows the ratio of the internal to external field as a function of wall thickness. The quantity B_i was measured at the center of the pipe ($z = 0.0$ mm, $r = 0.0$ mm). The plot is consistent with the estimate from Eq. 3 that wall thickness should exceed 5.0 mm. The dashed red line shows the lowest possible value of the field ratio, determined by in a run with a numerically-infinite value of relative magnetic permeability ($\mu_r = 10000.0$).

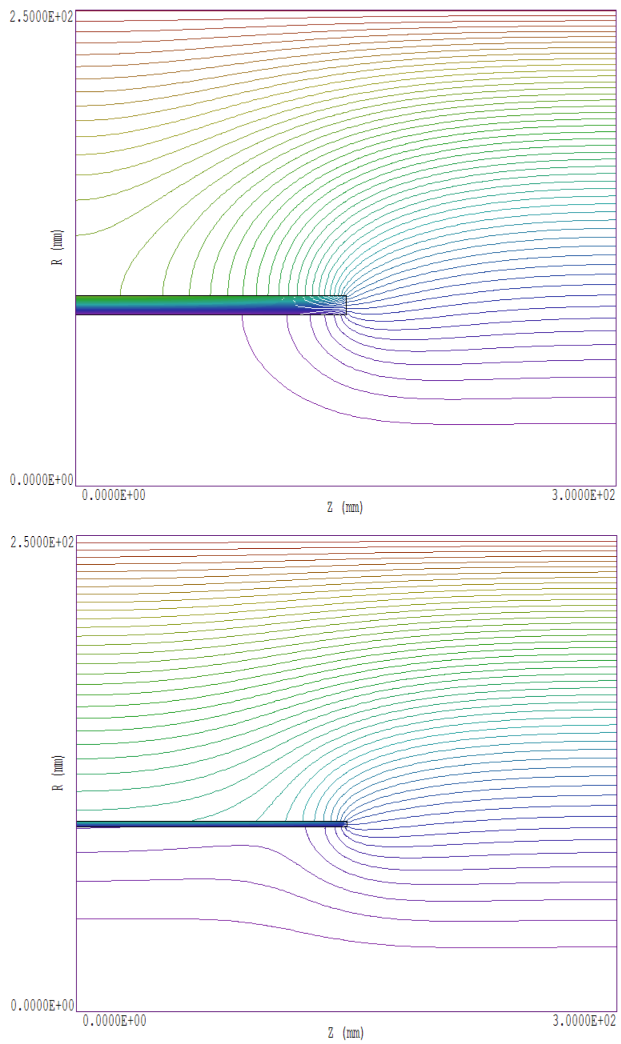

Figure 3: Magnetic shielding by a steel pipe: z-r plot of lines of $\mathbf{B}$. The calculation covers half the pipe with a symmetry boundary at $z = 0.0$. Top: wall thickness 10.0 mm. Bottom: Wall thickness 3.0 mm.

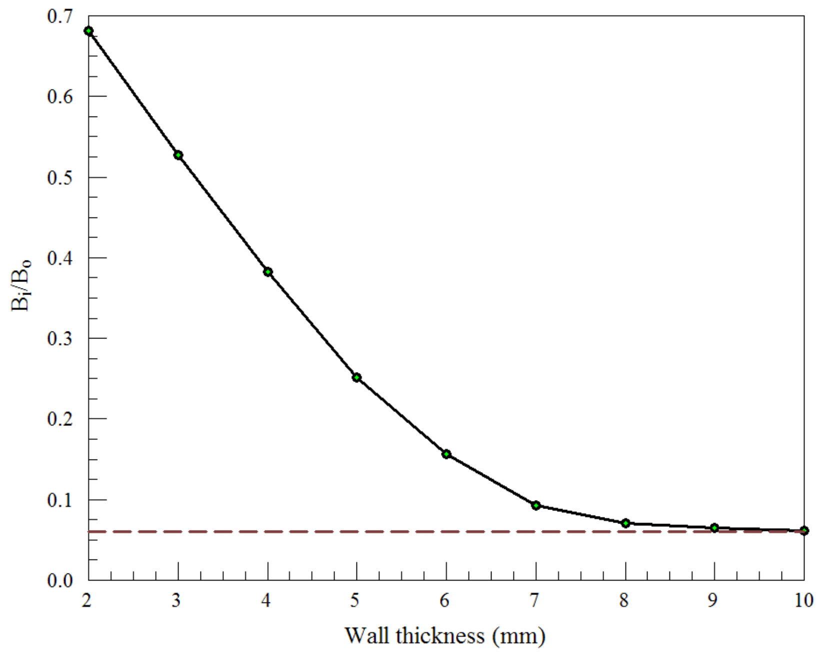

Figure 4: Ratio of internal to external field at the center of a steel pipe of radius $R_o = 100.0$ mm as a function of wall thickness. $B_o = 0.05$ tesla.

4 Three-dimensional shield calculations

The **Magnum** program can handle material saturation effects in shield calculations. Nonetheless, we must recognize that a nonlinear three-dimensional calculation requires a significant effort. Furthermore, the issue of scale disparities for thin material sheets is more acute. There is a strong motivation to plan calculations carefully to avoid wasted effort.

The most obvious path to minimize effort is to avoid saturation altogether. We saw in the previous section that shielding is ineffective when the material becomes saturated. Unless there is a severe weight constraint, you should use sufficient shielding material. Here is a recommended procedure for a three-dimensional calculation:

> Estimate an initial shield thickness from Eq. 3. Set up a linear calculation with a representative fixed μ_r for unsaturated material, applying the scaling technique discussed in Sect. 2 if necessary.
>
> Use **MagView** plots to check values of $|\mathbf{B}|$, ensuring that the flux density is less than about 1.6 tesla in all regions.
>
> If necessary, increase the material thickness and make additional calculations.

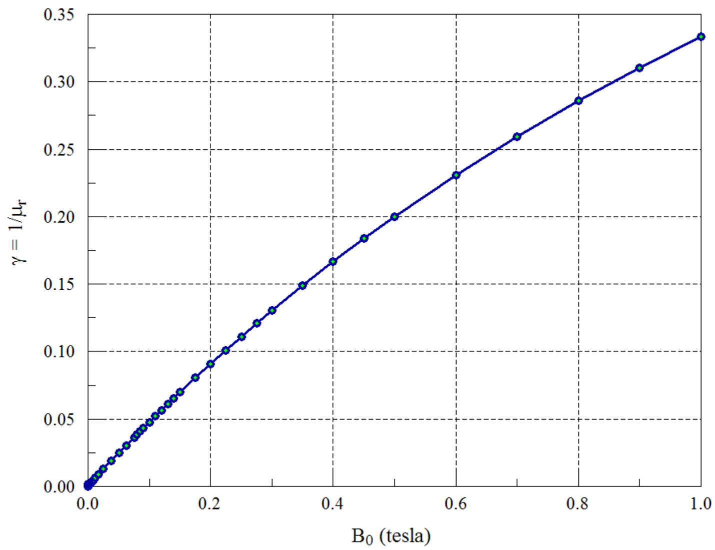

Figure 5: Saturation curve for type 50H450 steel, plot of $1/\mu_r$ versus the applied flux density in the material.

> If the shield ratio B_i/B_o is determined mainly by penetrations in the housing, the exact choice of μ_r has a relatively small effect.

The example `SHIELD_SATURATION` illustrates how to do a full nonlinear calculation with **Magnum**. The shield is a steel box with wall thickness 4.0 mm immersed in a uniform field $\mathbf{B} = B_0\mathbf{z}$, where $B_0 = 0.05$ tesla. The box has full widths 200.0 mm in x and y and 320.0 mm in z. The material has the saturation curve of Fig. 2. For a **Magnum** calculation, it is necessary to express the relative magnetic permeability in terms of the applied magnetic flux density. The resulting variation of $\gamma = 1/\mu_r$ is shown in Fig. fig:materialmagnum. The curve is relatively simple compared to the variation of Fig. 2. As a result, nonlinear problems in **Magnum** generally converge quickly.

Table 1 shows the **Magnum** control script `SHIELD_SATURATION.GIN`. The main difference from the **PerMag** setup is in the definition of the uniform background field. **Magnum** performs a scalar calculation and does not make use of the magnetic vector potential. Instead, the *BUni* command is used to set a uniform value of applied field for the reduced potential calculation. In this case, it is necessary to set symmetry conditions on the upper and lower boundaries in the direction of the field (Region 3). One of the boundaries is located at the symmetry plane $z = 0.0$ to reduce the run time of the calculation. The natural boundary condition for **Magnum** (a perfectly-conducting wall) applies on the boundaries in x and y. By symmetry, it is sufficient to model only the first quadrant in the x-y plane. The field-

Table 1:

```
SolType = STANDARD
Mesh = Shield_Saturation
DUnit =   1.0000E+03
ResTarget =   1.0000E-07
MaxCycle =    1000
BUni Z 0.05
Omega 1.95
Avg 0.75 15
* Region 1: Air
Mu(1) =   1.0000E+00
* Region 2: Steel 50H450
Mu(2) = (Table, steel50H470_magnum.dat)
* Region 3: Boundaries along z
Potential(3) = 0.0
EndFile
```

exclusion condition on the outer boundaries applies to the field components created by the shield. There is a significant air volume around the shield to approximate the free-space condition. The *Avg* command controls the iterations of the nonlinear calculation. Figure 6 shows the magnitude and direction of **B** in the plane $y = 0.0$n mm. Note that the field level in the shield is clamped close to the saturation value. For the given wall thickness, the reduction factor is $B_i/B_o = 0.445$.

In this example, the box shape made it easy to represent the thin walls of the shield. For arbitrary shapes, it may be necessary to use scaling principles like those discussed in Sect. 2. A larger local element size may be employed with thicker walls. An approximate scaling technique can be applied in the presence of material saturation. Inspection of Fig. 6 shows that the flux density magnitude is almost uniform across the thickness of the shield at all positions. We would get approximately the same solution with the following procedure:

Double the thickness of the shield walls.

In the **PerMag** curve of B versus μ_r, halve the values of B.

Convert the curve to **Magnum** format by replacing the independent variable with $B_o = B/\mu_r$.

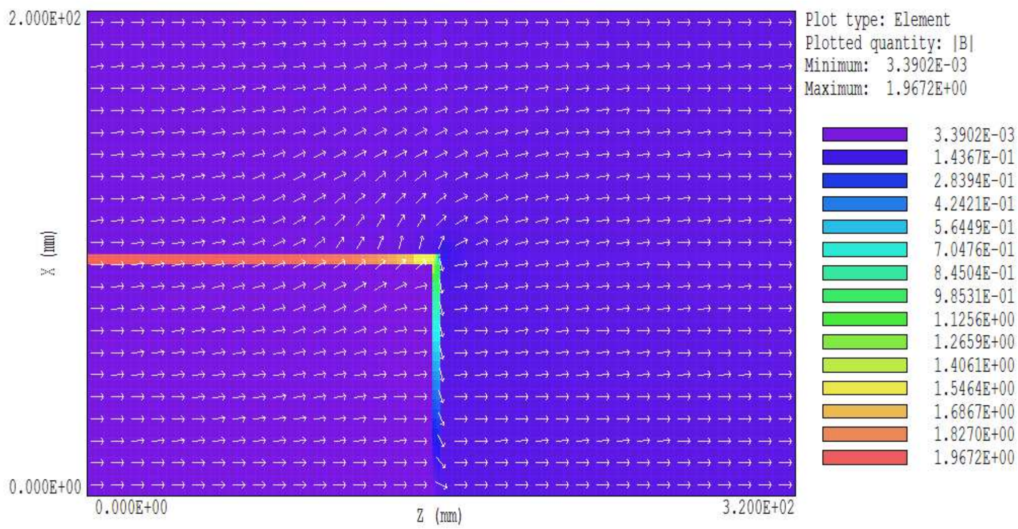

Figure 6: Saturated magnetic shield calculation with Magnum. Plot of $|\mathbf{B}|$ in the plane $y = 0.0$ mm.