



## **Inductance calculations with numerical methods**

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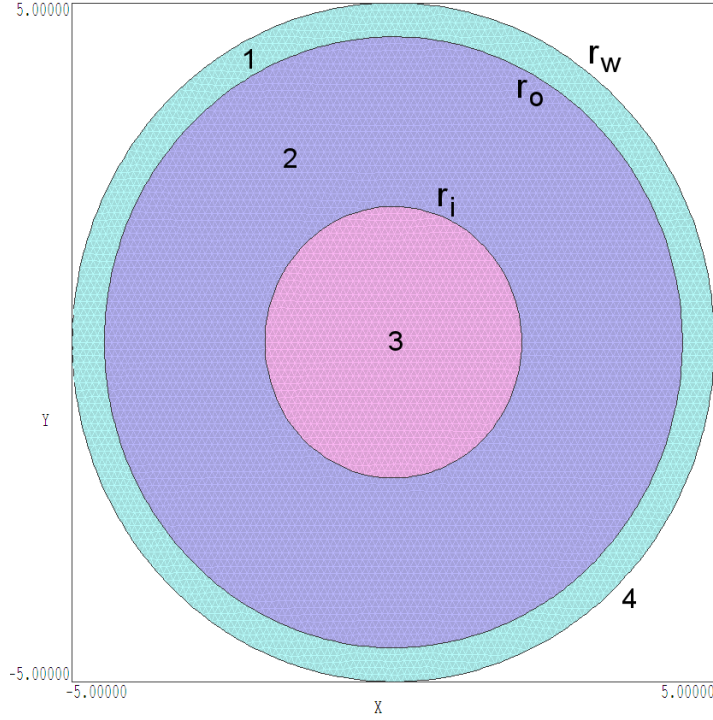


Figure 1: Model of a coaxial transmission line in the low-frequency limit, showing dimensions and regions. 1) Outer conductor. 2) Air. 3) Inner conductor. 4) Flux-confining boundary.

This tutorial illustrates how to use finite-element codes to calculate inductance. The calculations address transmission lines in the limit that the radiation wavelength is long compared to the transverse dimensions of the structure. I will show how to set up solutions with the two-dimensional **PerMag** code (for static magnetic fields) to get useful information for both the low and high-frequency limits.

We begin with a simple example that can be compared to theory, the coaxial transmission line shown in Fig. 1. Current flows out of the page through the center conductor. A return current flows into the page through the outer conductor. With the assumption that there are no ground loops, the drive and return currents are equal in magnitude; therefore, there is no magnetic field outside the assembly. The center conductor has radius  $r_i$ , while the outer conductor has inner radius  $r_o$  and outer radius  $r_w$ .

In the low frequency limit, the current is uniformly distributed over the area of the inner and outer conductors to minimize the circuit resistance. In this case, it is easy to set up a **PerMag** solution. The three volumetric regions (inner conductor, air, outer conductor) have the material property  $\mu_r = 1.0$ . The inner conductor carries total current  $I_0 = 1.0$  A and the outer

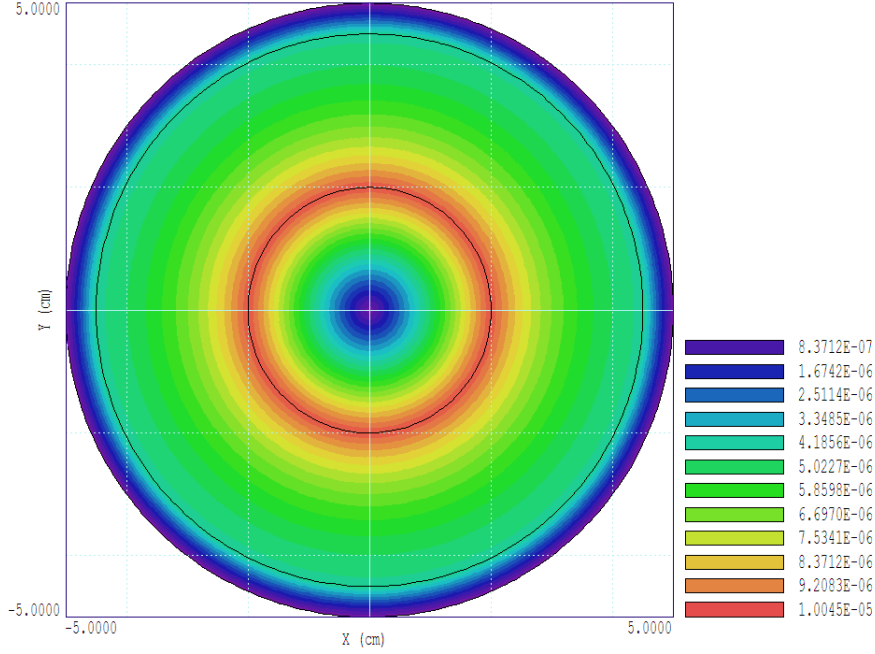


Figure 2: Coaxial transmission line in the low-frequency limit, plot of  $|\mathbf{B}|$  for a drive current  $I_0 = 1.0$  A.

conductor has current -1.0 A. **PerMag** automatically apportions current to elements of a conducting regions to represent a uniform current density. Figure 1 shows the geometry with  $r_i = 2.0$  cm,  $r_o = 4.5$  cm and  $r_w = 5.0$  cm. Region 1 is the outer conductor, Region 2 is air and Region 3 is the inner conductor. It is necessary to specify a condition on the outer boundary (Region 4). We set the vector potential  $A_z$  equal to zero, equivalent to the condition that flux is contained inside (*i.e.*,  $\mathbf{B}$  is parallel to the surface). The element size approximately 0.1 cm. The plot of  $|\mathbf{B}|$  in Fig. 2 shows that the magnetic flux density permeates the conductors. The predicted magnetic flux density on the surface of the inner conductor is

$$B_\theta = \frac{\mu_0 I_0}{2\pi r_i} = 10^{-5} \text{ (tesla)}. \quad (1)$$

We can find the inductance per unit length in  $z$  ( $L_z$ ) by calculating the magnetic field energy per length ( $U_z$ ) and employing the relationship:

$$U_z = \frac{L_z I_0^2}{2} \quad (2)$$

**PerMag** has an automatic routine to integrate the magnetic field energy density

$$u = \frac{B^2}{2\mu_r\mu_0} \quad (3)$$

over the cross-section of the line. The resulting value,  $U_z = 1.09718 \times 10^{-7}$  J/m, implies that  $L_z = 2.1943 \times 10^{-7}$  H/m. The calculation shows that 22.8% of the energy is in the center conductor, 73.9% in the air space and 3.3% in the outer conductor.

The more familiar example is the high-frequency case where the magnetic field is excluded from the metal center and outer conductors. In this case, the inductance per length given by

$$L_z = \frac{\mu_r\mu_0}{2\pi} \ln(r_o/r_i). \quad (4)$$

The value for an air-filled transmission line with  $r_i = 2.0$  cm and  $r_o = 4.5$  cm is  $1.6219 \times 10^{-7}$ . This quantity is 73.9% of the low-frequency value because there no field energy in the conductors. In a **PerMag** calculation, a perfect conductor is a surface of constant vector potential. In this case, lines of magnetic flux density run parallel to the surface. I modified the solution of Fig. 1. Region 1 extends from  $r_i$  to  $r_o$  with  $\mu_r = 1.0$ , Region 2 is the outer boundary at  $r_o$  with  $A_z = 0.0$  tesla-m and Region 3 in the inner conductor of radius  $r_i$  with a fixed value  $A_z = A_{z0}$ . I made the arbitrary choice  $A_{z0} = 1.0$  tesla-m. The specifications lead to a solution with the desired properties: the magnetic flux density satisfies Eq. 1 in the air space and equals zero everywhere else.

In order to determine inductance, it is necessary to find the current flowing in the center conductor corresponding to choice of  $A_{z0}$ . For this we can utilize Ampere's law,

$$\oint \left( \frac{1}{\mu_0} \right) \mathbf{B} \cdot d\mathbf{l} = I_0. \quad (5)$$

The circuit integral is applied along a closed path in the air space surrounding the center conductor to find the total current  $I_0$ . To speed up this type of calculation, I created a configuration file `permag_inductance.cfg` using `permag_standard.cfg` as a template. It contains the definitions:

```
VOLUME
  Energy = &Bxz &Hxz * &Byr &Hyr * + 0.5 *
END
SURFACE
  IEnclosed = &Bxz $IMu0 *;&Byr $IMu0 *
END
```

The volume integral gives the total field energy per length, while the surface integral implements Eq. 5. I also prepared an analysis script with the content:

```

CONFIGURATION \fieldp\tricom\permag_inductance.cfg
OUTPUT Inductance02.DAT
INPUT Inductance02.POU
VOLUMEINT
LINEINT -2.5 -2.5 2.5 -2.5
LINEINT 2.5 -2.5 2.5 2.5
LINEINT 2.5 2.5 -2.5 2.5
LINEINT -2.5 2.5 -2.5 -2.5
ENDFILE

```

Under control of the script, **PerMag** sets the proper configuration file, loads the second solution file and opens a file *Inductance02.DAT* to record the results. The program performs the volume integral and takes four line integrals that constitute a closed path around the center conductor. The field energy is  $3.0822 \times 10^6$  J/m while the sum of current from the parallel component of the line integrals is  $6.1656 \times 10^6$  A. Applying Eq. 2, the inductance is  $L_z = 1.6263 \times 10^{-7}$  H/m, within 0.2% of the predicted value.

An important questions is when to apply the low or high frequency inductance values. Suppose the center and outer conductors in Fig. 1 have resistivity  $\rho$ . The resistance per unit length of the structure is

$$R_z = \rho \left( \frac{1}{\pi r_i^2} + \frac{1}{\pi(r_w^2 - r_o^2)} \right). \quad (6)$$

If the conductors in Fig. 1 are aluminum ( $\rho = 2.65 \times 10^{-8}$   $\Omega$ -m), then  $R_z = 3.89 \times 10^{-5}$   $\Omega$ /m. The transition from low to high frequency operations occurs near the frequency

$$f_0 = \frac{1}{2\pi(L_z/R_z)} = 38 \text{ Hz}. \quad (7)$$

Magnetic flux is excluded from the conductors and Eq. 4 holds for frequencies in the range  $f \gg f_0$ .

Of course, a numerical approach is not necessary with a geometry as simple as that of Fig. 1. Numerical methods offer a real advantage if the center conductor is non-circular. In this case, an analytic solution would involve derivation and evaluation of a series expansion, which usually involves considerably more work than a direct numerical approach. Another practical application would be calculation of the inductance with a displaced center conductor. For example, we may want to find the effect of sag in a long, air insulated line.

The second example is the two-wire transmission line shown in Fig. 3. Two conductors of radius  $a = 0.15$  cm separated by distance  $D = 1.0$  cm carry equal and opposite currents. If the wires are located in infinite free

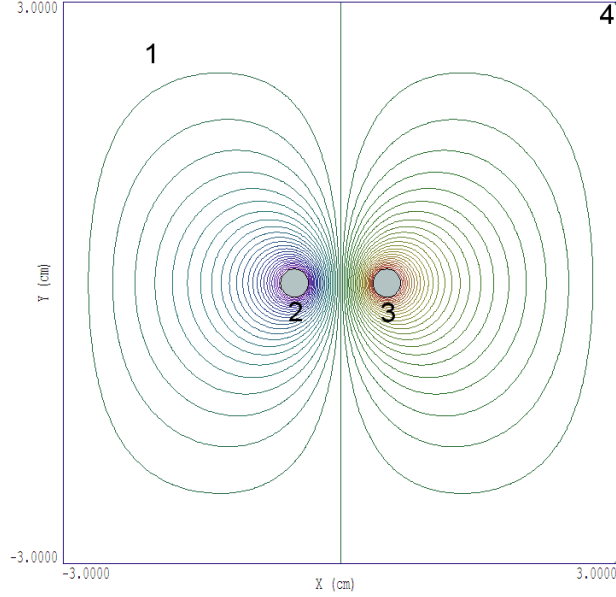


Figure 3: Lines of magnetic flux density, two-wire transmission line in a bounded space.

space, the analytic value for inductance per length in the high-frequency limit is

$$L_z = \frac{\mu_0}{\pi} \cosh^{-1} \left( \frac{D}{2a} \right) = 7.497 \times 10^{-7} \text{ (H/m)}. \quad (8)$$

One problem that we face in a numeric calculations is that finite-element solutions are performed in a finite space. The issue illustrates a general rule: it is often difficult to replicate an ideal analytic result with a numeric code. On the other hand, it is relatively easy to generate practical results. No real-world system is infinite, and our concern is frequently the perturbing effect of surrounding structures.

The first solution I created to approximate the infinite-space results had the geometry of Fig. 3. The solution area is a square box with 6.0 cm sides. The mesh is defined with fine elements near the wires and coarse elements in the surrounding volume. Region 1 is air, Region 2 is the left-hand wire with the condition  $A_z = -1.0$  tesla-m and Region 3 is the right-hand wire with the condition  $A_z = -1.0$  tesla-m. We must specify a condition on the boundary (Region 4). There are two choices in a standard finite-element solution: 1) Dirichlet condition where  $A_z = A_{z0}$  and lines of  $\mathbf{B}$  are parallel and 2) Neumann condition where  $\partial A_z / \partial \mathbf{n} = 0.0$  and lines of  $\mathbf{B}$  are normal to the surface. The second choice is clearly non-physical. The better choice is to set  $A_z$  equal to constant value. In this case, confinement of flux inside the box will introduce a shift in  $L_z$  from the infinite-space value.

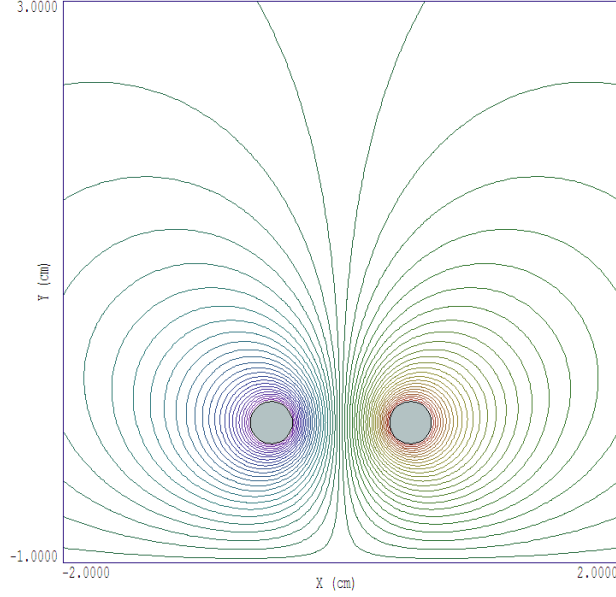


Figure 4: Magnetic field lines of a two-wire transmission line above a metal plate.

I used the same analysis techniques as those in the previous example. A area integral is used to find the magnetic field energy per length, while the current carried by the right-hand wire is determined from a circuit integral (Eq. 5). The energy is  $U_z = 2.7236 \times 10^6$  J/m and the current is  $I_0 = 2.7325 \times 10^6$  A. The corresponding inductance is  $L_z = 7.2954 \times 10^{-7}$  H/m, about 3% lower than the free-space value. To check the effect of the boundary, I created a second solution with an expanded volume (sides of length 12.0 cm). Here, the calculated inductance is  $L_z = 7.477 \times 10^{-7}$  H/m, a difference of only 0.3% from the theoretical value.

A practical application demonstrates the advantage of the numerical approach. Suppose we want to find how the inductance per length of the line changes if it passes near a metal object. For example, what happens if we run a television antenna cable across a metal roof? Consider placing the transmission line of Fig. 3 1.0 cm above a metal plate. To construct solution, we simple move the lower flux-excluding boundary from  $y = -6.0$  cm to  $y = -1.0$  cm. Figure 4 shows the resulting distribution of lines of magnetic flux density. The plate compresses lines in the lower part of the volume. Using calculated values of the field energy and wire current, the inductance per unit length is  $L_z = 7.0546 \times 10^{-7}$  H/m, a reduction of 6% from the free-space value.