

FINITE-ELEMENT CODES TO MODEL ELECTRICAL HEATING AND NON-LINEAR THERMAL TRANSPORT IN BIOLOGICAL MEDIA

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ABSTRACT

ETherm is a versatile finite-element software system to model heating in biological media for electrosurgery and other medical applications. The electrical field component calculates penetration of RF radiation into conductive dielectrics. The thermal solver finds time-dependent or steady-state solutions using stable diffusion methods with automatic time step adjustment. An important feature is the capacity to treat non-linear diffusion with temperature-dependent thermal properties such as blood perfusion to represent physical changes of tissues. The program also evaluates Arrhenius damage integrals by maintaining temperature integrals over tissue elements.

INTRODUCTION

The ETherm program models RF heating and thermal transport in biological media with emphasis on electrosurgery applications. The finite-element program handles two-dimensional systems with planar or cylindrical symmetry. ETherm simultaneously performs boundary value solutions for RF electric fields and an initial value solution for the temperature. A goal in the development of the program was effective use by researchers who may not be experts in numerical methods. The documentation and program interfaces allow an occasional user to control the solution process from mesh generation to analysis. The program runs on widely available personal computers.

This paper gives a brief description of mathematical methods used in ETherm. The following section describes the motivation for the finite-element approach and the organization of the program. An important feature is forward compatibility with the growing body of data on thermal properties of tissues¹. The program can organize a variety of input information including temperature-dependent material properties as well as heat source and electrode voltages with arbitrary time variations. The next section covers principles of the RF

field calculation in media with dielectric and resistive properties in the frequency regime of interest for electrosurgery.

Here, the electric field can be derived from a quasi-static potential. The field distribution determines a spatially dependent heat source term for the thermal calculation. When there are temperature-dependent volume resistivities the field distribution must be periodically updated. The following section describes the treatment of thermal transport on an arbitrary triangular mesh. The finite-element equations follow from application of conservation of energy over small volumes. The effects of blood perfusion are included in a form equivalent to that in the bioheat equation. An adaptation of the Dufort-Frankiel method² ensures numerical stability independent of the choice of time step. Although this method is absolutely stable on a rectilinear mesh, numerical divergence sets limits on the shapes of elements in a triangular mesh. The final section illustrates code capabilities with benchmark results.

COUPLED FINITE ELEMENT CALCULATIONS

Numerical simulations usually involve the expression of continuous variations in a discrete form that can be solved by digital computers³. The approach in finite-difference calculations is direct conversion of partial differential equations using difference operators. The process is referenced to points on a spatial mesh. In contrast, the basic units in finite-element calculations are small divisions of the solution volume⁴. The application of integral relationships like Gauss' law or energy conservation over the elements leads to difference equations. In both methods, the end result is a large set of coupled linear equations that are straightforward to solve on computers. One advantage of the finiteelement process is that the integral method is not closely tied to a rectilinear mesh. This makes it easier to derive difference equations for generalized

meshes. Furthermore, the definition of elements facilitates assignment of material properties and the identification of boundaries. For example, finite element calculations give the correct discontinuity of electric fields on dielectric surfaces. Another advantage is that all unspecified boundaries automatically assume a Neumann condition.

The two-dimensional calculations of ETherm are performed on a conformal triangular mesh like that of Fig. 1. The space is divided into triangular volumes so that element sides lie close to material boundaries. In comparison to rectilinear meshes, the division of Fig. 1b gives a good representation of curved boundaries where accuracy may be critical. The average element size can also be adjusted to enhance accuracy in regions of strong geometric variations. The mesh generation process starts with boundary input. ETherm includes a graphical drawing utility similar to popular CAD programs. The output of the program is a set of point, line and arc vectors that define the boundaries of regions (Fig. 1a). The mesh generator begins with a set of similar elements with known logical connections that fill the solution volume. Triangle vertices are shifted to conform to region boundaries. The affected vertices are labeled with the associated region number. For volume regions the program assigns the region number to elements inside the closed boundary. The result after smoothing is shown in Fig. 1b. The robust method is responsive to almost any user-generated geometry. Furthermore, the simple logical connections facilitate fast matrix inversions on personal computers.

Physical properties are associated with region numbers only during the solution process. ETherm uses the same mesh for the electrical and thermal solutions, reducing storage requirements. The regions of Fig. 1a illustrate the generality. For the RF solution, Region 1 is a conductive medium with $\epsilon_r = 1.0$ and $\rho = 13.6 \Omega\text{-m}$, Region 2 is a dielectric probe sheath with $\epsilon_r = 10.0$ and $\rho = \infty$. Region 3 is an electrode with a fixed potential of amplitude of 100 V and phase 0° , while Region 4 is a grounded outer wall. For the thermal solution Region 1 is tissue with $\rho = 1000 \text{ kg/m}^3$, $k = 20 \text{ W/m}^2\text{-s}$, and $C_p = 3500 \text{ J/kg-}^\circ\text{C}$. The dielectric sheath is a poor conductor with $k = 0.1 \text{ W/m}^2\text{-s}$. The electrode volume is an active region with a high thermal conductivity $k = 200 \text{ W/m}^2\text{-s}$.

It is straightforward to incorporate temperature-dependent material properties in the finite-element formulation. The main challenge is

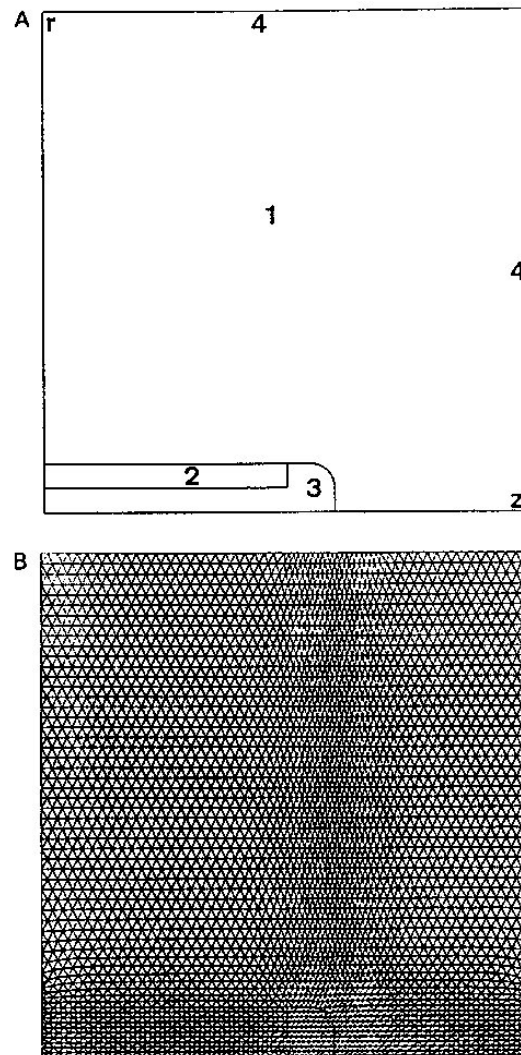


Figure 1

effectively organizing the diversity of possible combinations. A coupled electrothermal problem may require data on temperature variations of electrical resistivity, specific heat and thermal conductivity. In addition, it may be necessary to specify complex temporal variations of heat sources, surface temperatures or electrode voltages. The approach in ETherm is to store all material and temporal variations in an identical tabular format and pipe communication with the program through a standard interpolation unit. The user enters data through flexible format text files. Each data line contains a set of values for the independent and dependent variables. The program can store up to 32 tables with up to 1024 data lines. Table

storage can be flexibly assigned to any physical function through pointer references.

In coupled calculations thermal variations take place much more slowly than the electric field relaxation time. Therefore, it is sufficient to calculate the electric field distribution in quasi-equilibrium. The first steps in a coupled solution are to set values for electrical resistivity at ambient temperature and to determine the field by successive over-relaxation⁵. The volumetric heat source in an element with electric field amplitude E_i ; and conductivity σ_i is given by $\sigma_i E_i^2/2$. The RF distribution and any additional sources are used to initiate the thermal transport calculation. If there are no variations of electrode voltages and no temperature-dependent materials the initial RF distribution applies throughout the calculation. Otherwise, it is necessary to update the fields periodically. Program output consists of spatial distribution files of temperature, quasi-static potential, and material properties at specified times. For electrosurgery simulations ETherm maintains Arrhenius damage integrals for tissue elements. An interactive graphical postprocessor program uses the information to create a variety of plots. The user can also place up to 20 thermal and electrical probes inside the solution volume to record time variations in a history file.

RF ELECTRIC FIELD CALCULATION

For most electrosurgery simulations it is sufficient to reduce the Maxwell equations to a form similar to electrostatics. The criterion is that the system size L is much smaller than the wavelength of electromagnetic radiation at frequency f , or

$$\frac{Lf}{\sqrt{\epsilon\mu}} \ll 1. \quad (1)$$

For example, with $L = 0.10$ m, $f = 50$ MHz, $\mu = \mu_0$, and $\epsilon = \epsilon_0$, the ratio L/λ equals 0.05. In this limit inductive electric fields make a small contribution so that $\nabla \times \mathbf{E} \approx 0$. Therefore, the electric field can be expressed as the gradient of a scalar potential, $\mathbf{E}(x,t) = -\nabla\phi \exp(j\omega t)$.

The conductive current density is related to the electric field by $\mathbf{j}_c = \sigma \mathbf{E}$. The time-varying conductive current generates local concentrations of space charge, ρ_c . The equation for the conservation of conductive current is

$$\frac{\partial \rho_c}{\partial t} = j\omega \rho_c = -\nabla \cdot \mathbf{j}_c = -\sigma \nabla \cdot \mathbf{E}. \quad (2)$$

The Maxwell equation for the divergence of electric field is $\nabla \cdot (\epsilon \mathbf{E}) = \rho_c$. Combining this result with Eq. 2 gives

$$\nabla \cdot \left[\left(\epsilon - \frac{j\sigma}{\omega} \right) \nabla \phi \right] = 0. \quad (3)$$

Equation 3 is identical to the Poisson equation if we treat the quantity in parenthesis as a complex dielectric constant,

$$\epsilon = \epsilon' + \epsilon'' = \epsilon_r \epsilon_0 - j\sigma/\omega. \quad (4)$$

The equation for low-frequency fields in resistive media is the same as that for electrostatics except that the quantities ϕ_i and ϵ_i may be complex numbers. Physically, this means that there are phase differences in electric fields at different locations because the medium has both resistive and capacitive properties.

The finite-element equation for the potential at a vertex can be derived by applying Gauss' law on a surface surrounding the point the includes one-third the volume of each of the adjacent elements. The result for a two-dimensional planar geometry and the regular six element mesh used in ETherm is

$$\phi_o = \frac{\sum_{i=1}^6 W_i \phi_i}{\sum_{i=1}^6 W_i} \quad (5)$$

Equation 5 relates the value of the complex potential at a point to those at six neighboring vertices multiplied by complex coupling coefficients^{3,4} W_i . These numbers depend on the geometry and dielectric constant of elements adjacent to the line between the vertices. There is one relationship like Eq. 5 for each vertex. In a typical simulation the number of equations ranges from 5000 to 100,000. The large equation sets can be solved on typical personal computers in approximately a minute by the method of successive over-relaxation. This process consists of continually checking the deviation of potential values from the prediction of Eq. 5 and adding proportional correction factors⁵. Spatial derivatives of the complex potential give the magnitude and phase of electric field in elements of the solution volume.

THERMAL TRANSPORT

The finite-element equivalent of the bioheat equation⁶ is derived by applying conservation of energy over elements surrounding a vertex. The result is

$$\frac{\partial T_o}{\partial t} \sum_{i=1}^6 \frac{\rho_i C_{pi} a_i}{3} = \sum_{i=1}^6 W_i T_i - T_o \sum_{i=1}^6 W_i + \sum_{i=1}^6 \frac{S_i a_i}{3} + \sum_{i=1}^6 \frac{W_{bi} C_{pbi} a_i (T_b - \bar{T}_i)}{3} \quad (6)$$

where the sums are taken over elements surrounding a vertex with areas a_i , material densities ρ_i , specific heats C_{pi} and thermal conductivity k_i . The coupling coefficients are similar to those of the RF calculation except k_i replaces ϵ_i . In the final term representing blood perfusion the quantities W_b , C_{pb} and T_b are respectively the mass flow rate, specific heat and temperature of blood in the element region and \bar{T}_i is the average element temperature. There are several options for time differencing of Eq. 6. A good choice is the explicit Dufort-Frankiel algorithm² that preserves numerical stability for any choice of time step Δt . The form for a conformal triangular mesh⁷ is

$$T_o^{n+1} = T_o^{n-1} + \frac{t}{C_{pi} a_i} \left[\sum W_i T_i - \frac{T_o^{n+1} + T_o^{n-1}}{2} \sum \right] \quad (7)$$

$$\sum \frac{S_i^n a_i}{3} + \sum \frac{W_{bi}^n C_{pbi}^n a_i (T_b - \bar{T}_i^n)}{3}$$

where the superscript n denotes time $t = n\Delta t$ and the subscript i denotes the surrounding elements. At each step ETherm advances the temperature of all vertices with periodic correction of Δt to minimize run time and to preserve accuracy.

The Dufort-Frankiel method is absolutely stable when applied to finite-difference equations on a rectilinear mesh. We have observed a numerical instability that may grow in certain regions of conformal finite-element meshes. An analysis Eq. 7 and the forms of the coupling coefficients show that solutions are unstable in mesh areas where there are approximately eight or more

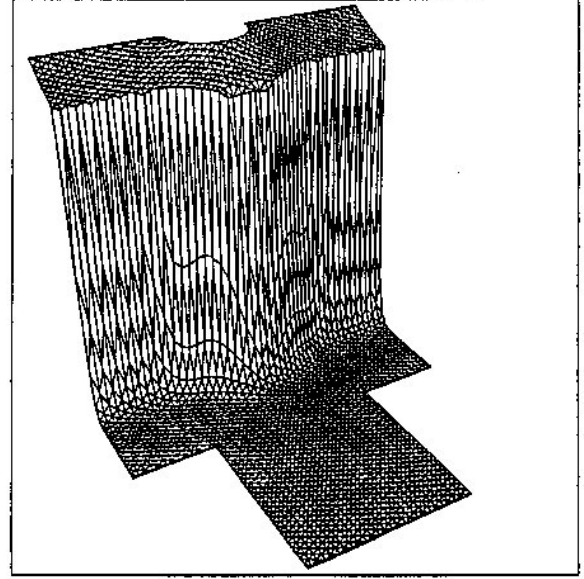


Figure 2

contiguous elements with an internal angle greater than 90° . Small changes in the element geometry can stabilize the divergence. ETherm checks all triangles before a calculation and warns of potential problem areas.

BENCHMARK TESTS

Two examples will serve to illustrate some capabilities of ETherm. The first is a challenging non-linear problem, propagation of a thermal bleaching wave. This sharp temperature discontinuity, similar to a shock wave, can occur in a material where the thermal conductivity rises with temperature. Figure 2 shows a wireframe plot of temperature in a medium where the thermal conductivity jumps by a factor of 100 within a 2° span near 20° . In the solution, a temperature rise from 0° to 25° in 0.1 ms on the top-left boundary creates a transition wave that propagates downstream. The figure shows the temperature distribution at 3.5 ms for an ambient thermal conductivity of $k = 1.0 \text{ W/m}^2\text{-s}$.

Figure 3 shows results for combined RF heating and thermal transport for the system of Fig. 1. Figure 3a is a contour plot of quasi static electric potential for frequency $f = 200 \text{ MHz}$ at reference phase 0.0° . The displacement current across the dielectric sheath is large at the high frequency so that the assembly probe acts like a metal cylinder with radius equal to the sheath outer radius. The total power transferred to the medium is 7.278 W. At lower frequency (10 MHz) heating is concentrated at the probe tip and the power transfer

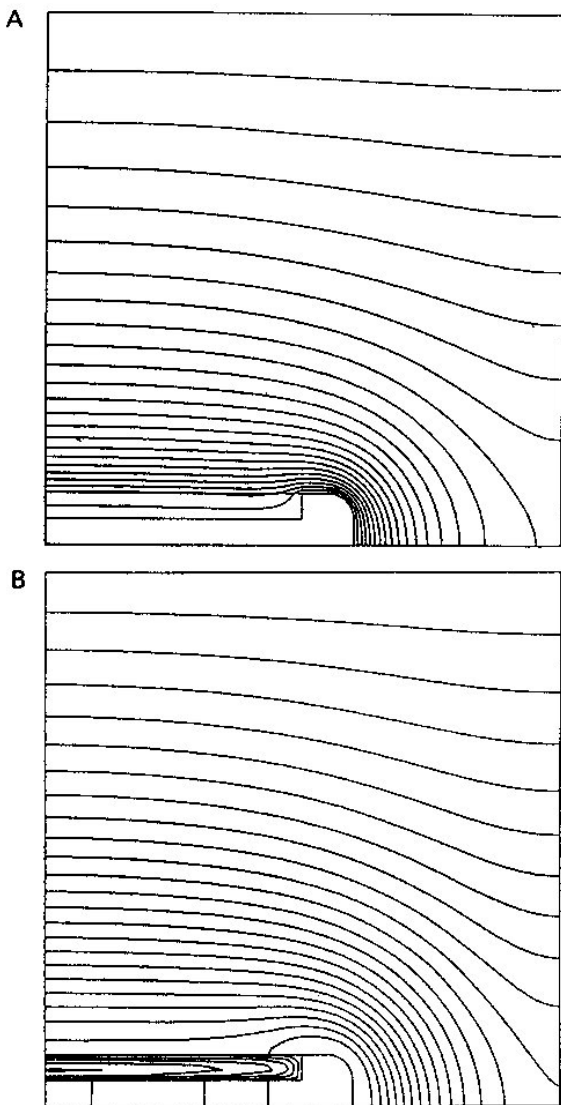


Figure 3.

drops to 4.035 W. Figure 3b shows isothermal lines at time $t = 3.5$ ms after initiation of the probe voltage. The solution determines the effect of probe heat conduction on the maximum tissue temperature.

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