Field Precision LLC

Simulation of a pulsed dipole radiator

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An intense radiation pulse occurs when two oppositely-charged high-voltage electrodes connected by a spark gap are short-circuited. This work described in this tutorial has two purposes:

Illustrate good parameter choices and setup techniques for **Aether**.

Confirm that the code is correctly representing the system physics.

I will work through the following benchmark calculation. Two metal hemispheres of radius 7.50 cm in a high-pressure gas are charged to ± 500.0 kV. They are separated by a narrow gap of width 0.5 cm, a portion of which rapidly changes from an insulator to a conductor (the switch). Figure 1 shows the simulation geometry. I took advantage of the system symmetry to minimize the run time. I treated one half of the assembly in the space $z > 0.0$ cm with a perfectly-conducting boundary in the plane $z = 0.0$ cm. An absorbing layer covered the outer boundaries. I choose an element size of 0.25 cm in the region of the hemisphere, 0.50 cm over the surrounding gas region, and 0.25 cm in the absorbing layer. Note that the vacuum volume was divided into two regions, a sphere of radius 17.0 cm centered at the origin and the remaining space out to the absorbers. The surface between the regions was used to generate plots. To summarize, the mesh contained the following regions:

Region 1. *OuterLayer*, a layer of thickness 0.25 cm on all boundaries except *ZDn*.

Region 2. *VacuumOut*, the vacuum volume except for Region 3.

Region 3. *VacuumIn*, a sphere of radius 17.0 cm centered on the origin.

Region 4. *Hemisphere*, the charged electrode.

Region 5. *CondBound*, a metal sheet of thickness 0.25 at $z = 0.0$ cm.

Switch 6. *Switch*, a cylindrical region of radius $R_s = 0.5$ cm and length $L_s/2 = 0.25$ cm that connects the electrode to the conducting boundary.

An electrostatic solution with **HiPhi** was required to set the initial state of electric fields for the charged electrodes. For this solution, the outer layer and the conducting sheet were set to a fixed-potential value of 0.0 V. The hemisphere was at potential $V = 500.0$ kV. All other regions were dielectrics with $\epsilon_r = 1.0$. Figure 2 shows the resulting equipotential lines. The fields in the gap matched the initial state of the electromagnetic solution, but there were deviations from the ideal free-space solution at large distances. The
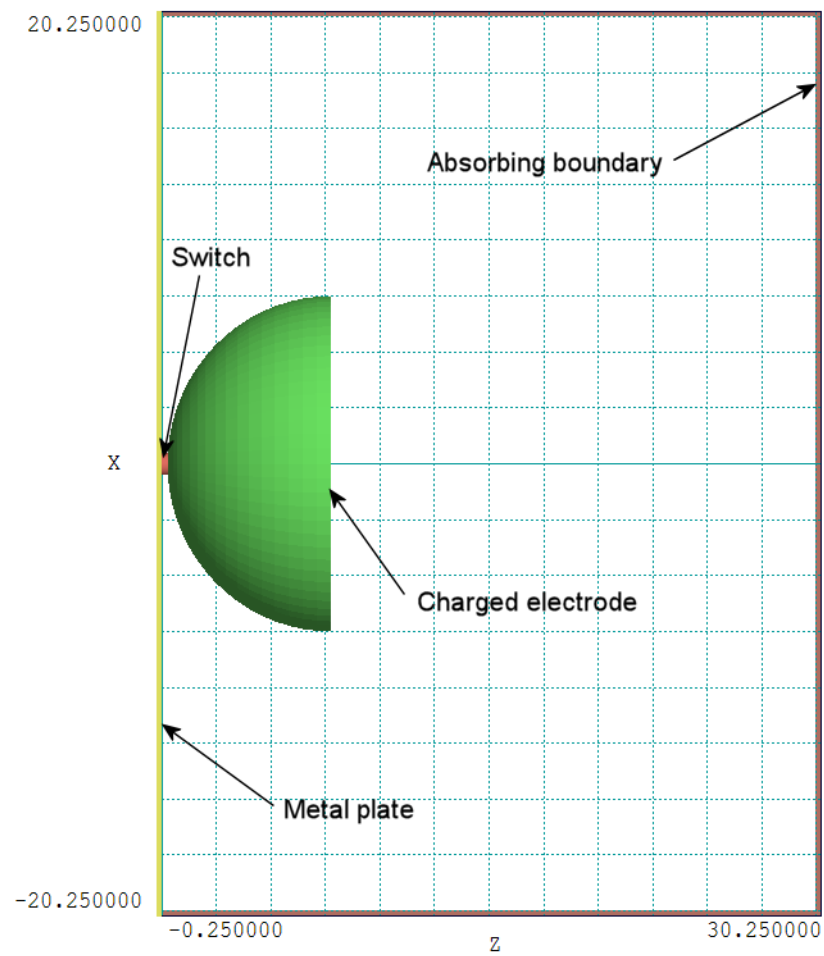
Figure 1: Geometry of the benchmark system, dimensions in cm.

discrepancies were not critical because the far fields represented only a small fraction of the field energy. The main effect was to introduce DC offsets in the electric-field probe signals. The stored energy calculated by **PhiView** for the solution of Fig. 2 plus its reflection was 5.278 J. For a total voltage difference $V_0 = 1.0$ MV, the capacitance of the assembly was $C = 10.53$ pF. To estimate the inductance, I created a magnetostatic solution with **Magnum**. In this solution, a current of 1.0 A flowed along the axis from $z = 7.5$ cm to $z = -\infty$. The outer layer, hemisphere, conducting boundary and switch had the property $\mu_r = 0.0001$ to represent perfect conductors. The magnetic field energy in the vacuum regions and their reflections was 1.417×10^{-9} J, giving an inductance $L = 28.34$ nH. For comparison, the inductance of the switch region is given approximately as

$$L = \frac{\mu_0}{2\pi} \ L_s \ \ln\left(\frac{R_0}{R_s}\right). \tag{1}$$

Taking $R_0 = 0.2$ m, the predicted inductance value is $L = 3.68$ nH.

After the spark gap switches, the system approximates an LC oscillator. The predicted frequency is

$$f_0 = \frac{1}{2\pi\sqrt{(LC)}}. \tag{2}$$

Inserting values, the estimated resonant frequency is 291 MHz. We can obtain a more accurate value using **WaveSim**. This program has the capability to determine resonances of open or highly-damped structures. Figure 3 shows the geometry, a figure of revolution about the axis at the bottom. Again, the hemispheres had radius 7.5 cm and a gap spacing of 0.5 cm. The 0.25 cm absorbing layer on the outer boundary simulated the free space condition. The system was excited by the small axial current source shown. The quantity rH_θ was measured by a probe at $r = 12.0$ cm. The first step is to perform a resonant scan of the frequency space. A sharp resonances does not occur because of the strong radiation damping. Figure 4 shows the probe response as function of frequency over the range 0.2 GHz to 1.2 GHz. The TM_{010} resonance occurred at near 450 MHz. The second step was to search for a resonance solution, zeroing in on the peak. The best value was $f_0 = 437.9 MHz$ ($\omega_0 = 2.751 \times 10^9$ s^{-1}, $\lambda = 0.686$ m). The resulting electric field lines are plotted in Fig. 3.

I chose a shorted switch conductivity for a relatively low resistance. In this case, damping of the LC oscillation resulted mainly from radiated energy. The radiation resistance of a Hertzian dipole of length dl is

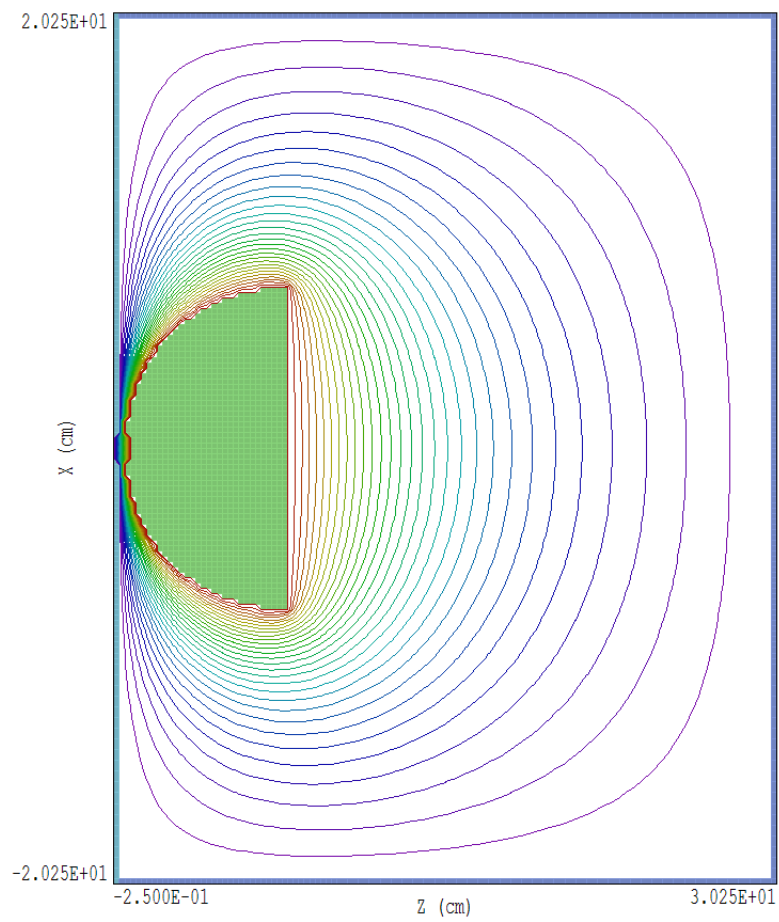$$R_r = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2. \tag{3}$$

Figure 2: Equipotential lines of the electrostatic solution in the plane $y = 0.0$.
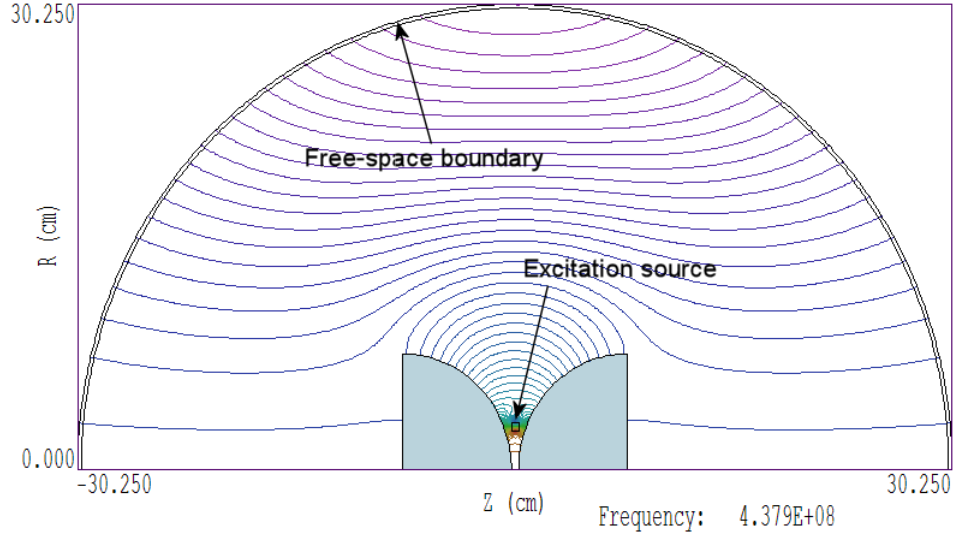
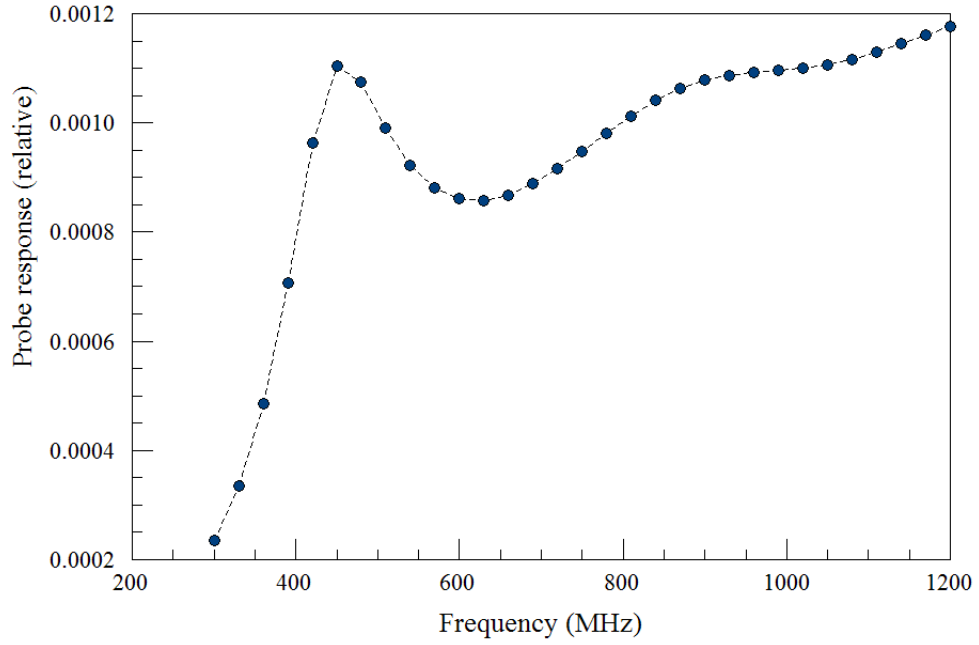Figure 3: Geometry for a **WaveSim** calculation of the dual-hemisphere system.

Figure 4: Probe response as a function of frequency, **WaveSim** calculation of the dual-hemisphere system.

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Taking $dl = 0.15$ m, Eq. 3 gives a value $R_r = 37.8$ Ω. The damping parameter for the LC circuit is

$$\beta = \frac{R_r}{2L} \tag{4}$$

For $L = 28.34$ nH, the damping parameter is $\beta = 6.67 \times 10^8$ s^{-1}. Because $\beta < \omega_0$, circuit was underdamped and we expect to see an oscillation that decays within a few ns. The electrical conductivity in the switch region was zero at $t = 0.0$ and rapidly increased to σ_0 S/m. I used a smooth step function that reached its full value in 0.5 ns. The switch had length $L_s = 0.0025$ m and cross-section area $A_s = 7.5 \times 10^{-5}$ m^2. The resistance in the on-state was

$$R_s = \frac{L_s}{\sigma_0 A_s} \tag{5}$$

The choice $\sigma_0 = 100.0$ S/m gives a switch resistance $R_s = 0.33$ Ω, a small value compared to the radiation resistance. The complete **Aether** input file is listed in Table 1. The simulation time of 10 ns was based on expected frequency and damping. Two notable features are the appearance of the *EInit* command to set initial electric field values from the **HiPhi** solution and the *SigMod* command to introduce a time-dependent switch conductivity. The run time was about 28 minutes.

Figure 5 shows the electromagnetic pulse moving out from the switch region and around the electrode. The quantity $|\mathbf{H}|$ plotted both on a sample slice at $y = 0.0$ cm and over the surface of the electrode. Note that the intervals for color coding are at logarithmic intervals. I added this feature to **Aerial** specifically for radiation calculations of this type. Figure 6 shows the time variation of B_θ at a position with radius 18.0 cm and polar angle 60°. As expected, there was a damped oscillation near the predicted resonant frequency. The time between zero crossings in the late-time signal was 2.46 ns, corresponding to frequency 454 MHz. Figure 7 shows the Fourier transform of the signal. There is a broad peak centered on the resonant frequency. Note that the fast current risetime in Fig. 5 and the high-frequency content in Fig. 7 were the result of rapid energy transfer in the high-field region close to the switch. The quantities depend on the choice of switching time and the dimensions of the switch region. To conclude, Table 2 shows a comparison of peak probe signals at different polar angles. The values are consistent with a dipole radiation pattern.

Table 1: Aether input script

```
* ---- CONTROL ----
Mode = Pulse
Mesh = Hemisphere
DUnit =   100.0
TMax =   10.0E-9
Dt = 5.0E-12
EInit Hemisphere
* ---- REGION PROPERTIES ----
* OuterLayer
AbsLayer(1) 0.25
* VacuumOut
Epsi(2) =  1.0
Mu(2) =   1.0
* VacuumIn
Epsi(3) =  1.0
Mu(3) =   1.0
* Hemisphere
Metal(4)
* CondBound
Metal(5)
* Switch
Epsi(6) = 1.0
Mu(6) = 1.0
Sigma(6) = 100.00
SigMod(6) = SmoothStep.MOD 0.5E-9 1.0
* ---- DIAGNOSTICS ----
DTime 0.5E-9
* Switch current
History = 0.125  0.125  0.125
* Switch gap
History =  2.50  0.00  0.375
* Far field probes (angles relative to dipole axis)
* 90.0 deg
History =  18.0000    0.0000       1.0000
* 60.0 deg
History =  15.5900    0.0000       9.0000
* 30.0 deg
History =   9.0000    0.0000      15.5900
*  0.0 deg
History =   0.0000    0.0000      18.0000
EndFile
```

Table 2: Peak values of $|\mathbf{H}|$ as a function of polar angle at a radius of 18.0 cm.

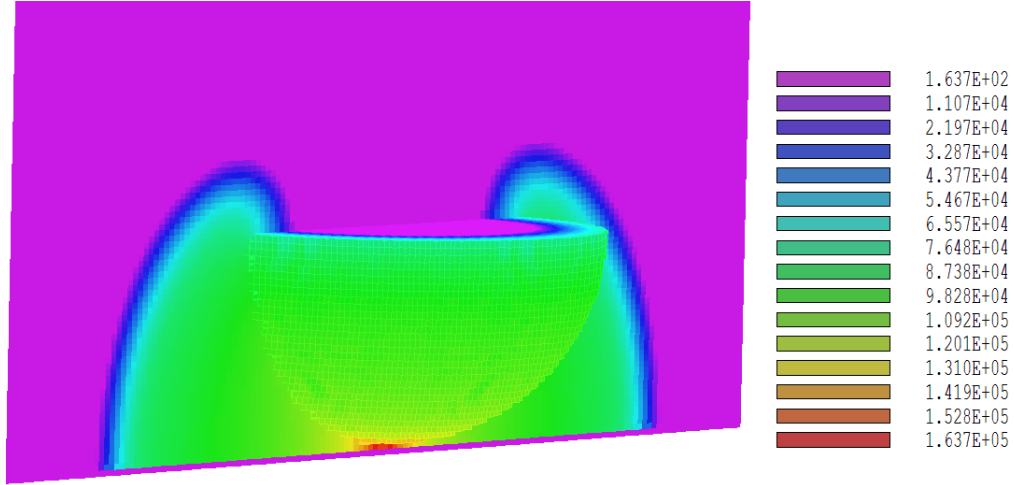| θ (deg)	$|\mathbf{H}|$(peak) (A/m)
86.8	4121.0
60.0	3761.0
30.0	2342.0
0.0	0.0

Figure 5: Three-dimensional plot of $|\mathbf{H}|(z, x)$ in the plane $y = 0.0$ cm at $t = 0.5$ ns.
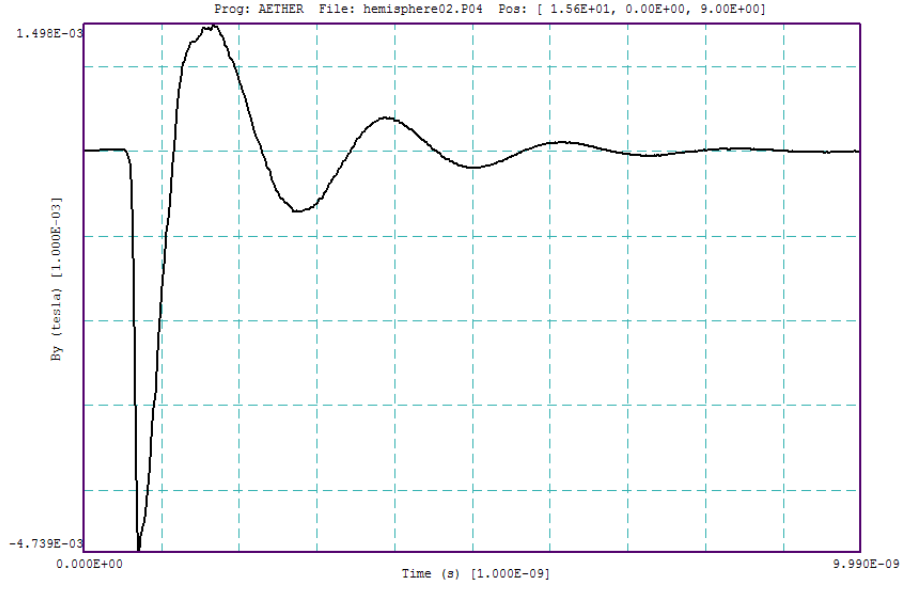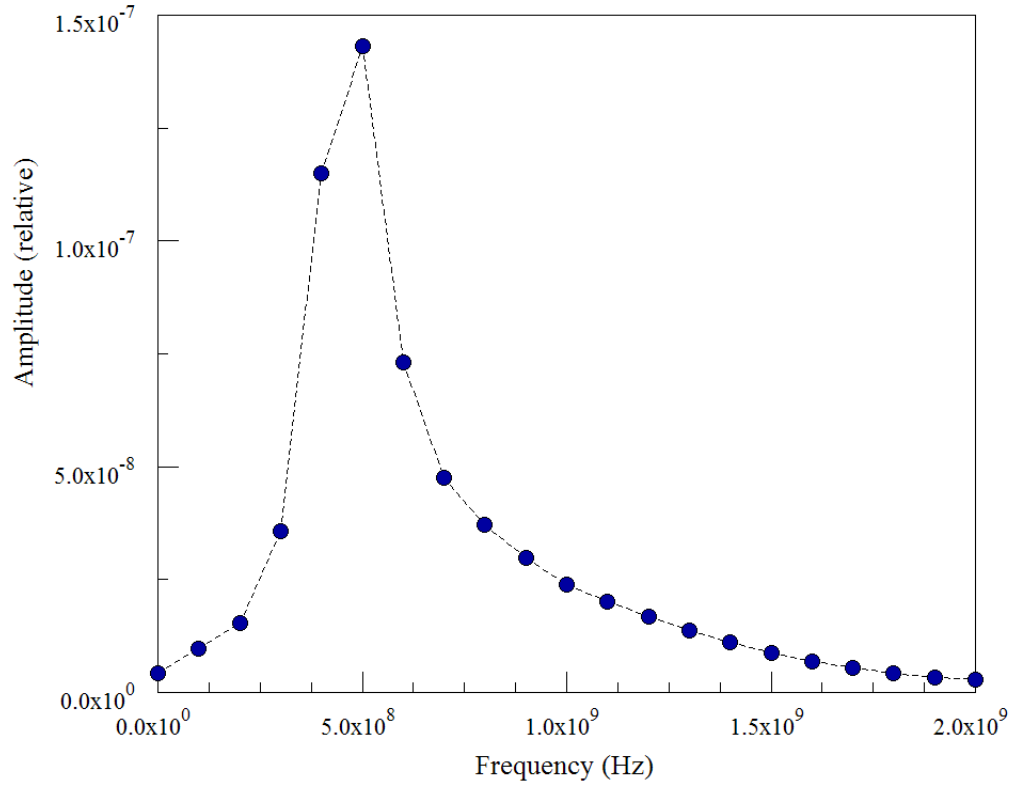
Figure 6: Plot of $B_\theta(t)$ at radius 18.0 cm and polar angle 60º.

Figure 7: Fourier transform of the signal of Fig. 6.