Numerical electrode design
via the equipotential method

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A frequent goal in high-voltage electrode design is a configuration that generates a field that is uniform over a working region and decreases moving out of the region. A familiar application area is pulsed gas lasers which should support a uniform discharge between long parallel plates. A simple radius on the plate edges causes field enhancement with the risk of sparking. The Rogowski[1] method is commonly applied to determine good shapes for electrode edges in a two-dimensional planar geometry. The procedure starts from analytic result, Maxwell's solution for a thin, finite-width plate above an infinite ground plane. Rogowski recognized that the field magnitude decreases moving away from the center along the equipotential line at about half the applied potential. Replacing the area above this line with a solid electrode leaves the field beneath unchanged. The general strategy of the equipotential method is to start with a generating electrostatic solution, to inspect equipotential lines to check for a desired property and then to use the line to determine the shape of an electrode to generate a working solution.

This tutorial shows how to carry out the process using numerical methods. While there are a limited number of analytic generating solutions, the numerical approach offers an unlimited set of possibilities with regard to symmetries, shapes and boundary conditions. This report addresses two-dimensional planar and cylindrical solutions. It gives a step-by-step description of the procedure with emphasis on **EStat** capabilities specially matched to the application. A following tutorial discusses a three-dimensional procedure using **HiPhi**.

We will begin with the standard Rogowski calculation of a plate above a ground plane. Figure 1 shows the geometry as represented in the **Mesh** *Drawing editor*. The planar solution volume has a Neumann boundary on the left side (y axis) to represent half of a plate above a ground plane. It could also represent one quarter of a system with upper and lower electrodes symmetric about the x axis. The upper section of Fig. 2 shows the starting dialog with boundaries chosen to approximate infinity. The simple regions are straightforward to draw with grid snap active:

- Region 1 is a rectangle that encompasses the solution volume.
- Region 2 is a line from (0.0,1.0) to (2.0,1.0).
- Region 3 consists of lines along the lower, right and upper boundaries. The left boundary assumes the default Neumann condition.

Region properties are assigned after the drawing is complete. The lower section of Fig. 2 shows the dialog settings. In the electrostatic solution, Region 1 has the properties of air or vacuum. The *Filled* setting means that the region number is assigned to both elements and nodes. The region definitions of the unfilled generating electrode and ground regions over-write selected nodes. The next action is to set the sizes of elements in the foundation mesh to high accuracy. Clicking the *Foundation display* button shows a representation of a default element size, 0.050 in both directions. Right click, pick the command *Divide zone X/Z* and set a dividing boundary at $x = 3.0$. Next, pick the command *Change element size X/Z*, highlight the left zone and enter 0.025. Similarly, set a division in the vertical direction at $y = 2.0$ and set the element size in the lower section to 0.025. The result is shown in Fig. 3. Click the *Foundation display* button again to return to the normal display. Save the script `RGInit.MIN`, process it and save the mesh file `RGInit.MOU`.

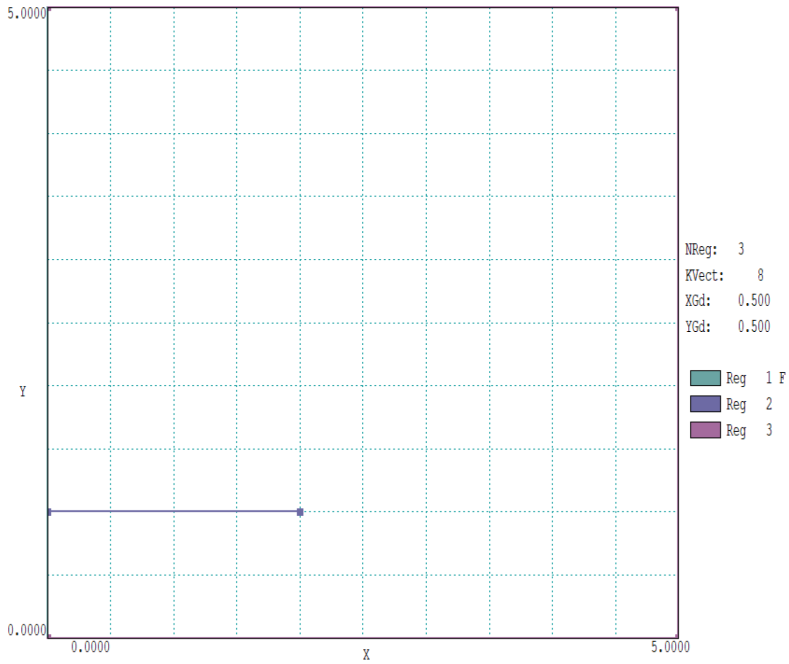

Figure 1: Generating solution geometry for Rogowski electrodes

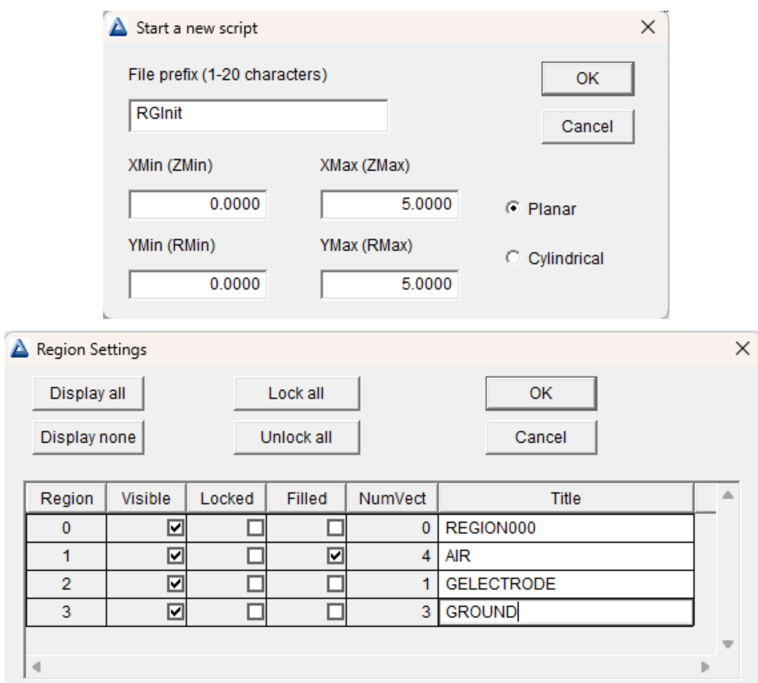

Figure 2: **Mesh** *Drawing Editor* dialogs. a) Start the drawing. b) Assign region properties

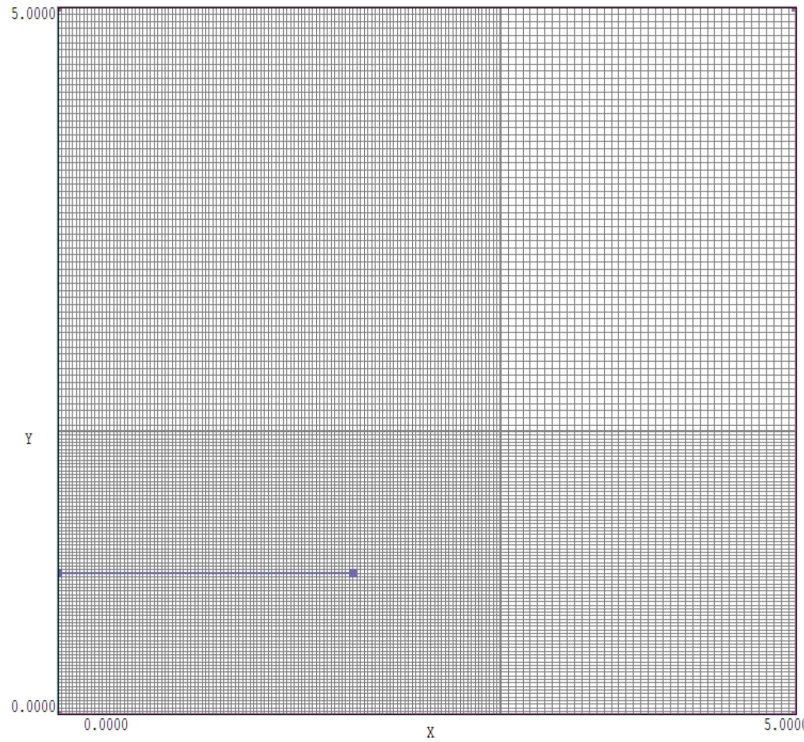

Figure 3: Variable mesh resolution in the generating solution for Rogowski electrodes

Run **EStat** to create the generating electrostatic solution. Click the *Setup solution* button and load the file created by **Mesh**. **EStat** analyzes the file content and displays the dialog of Fig. 4. The *Planar* setting means the solution varies in x and y and has infinite length in z. The interpretation of the dimensions in the **Mesh** file is set to centimeters for convenience – the conclusions we will draw are independent of units. The elements and node of the `AIR` region have $\epsilon_r = 1.0$. The node potential of the generating electrode is 1.0 V, so the electric field at the symmetry boundary at $x = 0.0$ cm is $|E| \cong 100$ V/m. Save the **EStat** script `RGInit.EIN`, click the *Run solution* button and process the input script to create the solution file `RGInit.EOU`.

Click the *Analyze solution* button and load `RGInit.EOU`. A variety of quantitative and qualitative operations can be performed in analysis menu. We will create contour line plots for this study. Zoom in on region around the electrode edge and create plots of contours of electrostatic potential ϕ and electric field magnitude $|\mathbf{E}|$. Figure 5 shows a superposition of the two plots. The equipotential lines are almost horizontal on the side near the symmetry axis. They curve around the end of generating electrode and loop back. The lines of field magnitude have a maximum value at the sharp end of the generating electrode and decrease moving away. The dashed blue line $y = 0.5$ cm intersects the midpoint of the gap between the generating electrode and ground at $x = 0.0$ cm. The point to note is that all equipotential lines that start below the line move into a region of decreasing $|\mathbf{E}|$ and lines that start above move through a region of increasing field magnitude before the field decreases as

[1]W. Rogowski and H. Rengier, Arch. Elekt. 16, 73 (1926).

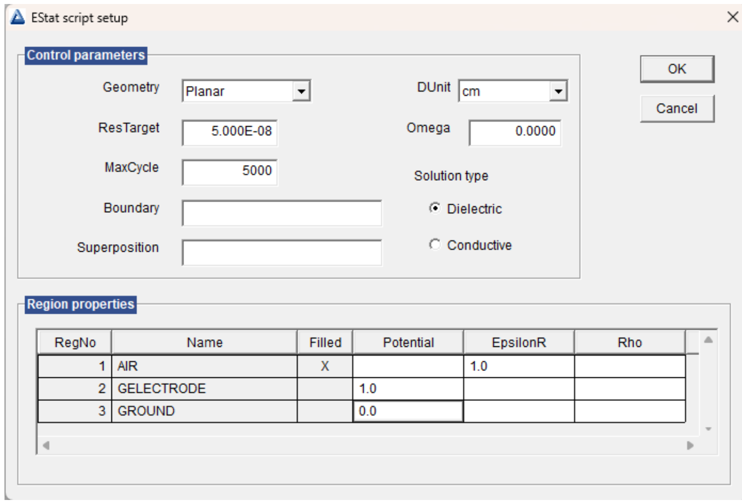

Figure 4:

they move around the generating electrode. The line that starts at $y = 0.5$ cm has the following favorable properties:

- The value of $|\mathbf{E}|$ decreases monotonically as in moves away from the symmetry axis.
- The line cover the maximum span with approximately uniform $|\mathbf{E}|$.

This equipotential line, the Rogowski profile, will be used as the basic for the working solution.

We need the coordinates of the equipotential line to create a mesh for the working solution. With `RGInit.EOU` loaded in **EStat**, pick *Analysis/Equiline tool* in the menu. Accept the default output data file name *RGInit.DAT*. In the dialog, set the potential value to 0.5 V and $NSkip = 5$. The program opens and writes a data record. Click the *Close data record* button, choose *File/Edit files* from the menu and load the data file. It consists of a long series of vectors with length on the scale of the fine mesh resolution. Moving down, there is a shortened list of vectors with lengths about five element widths in the format of line vectors for the **Mesh** program:

```
Reduced vector set for Mesh along potential contour
   Potential:  5.000000E-01   NSkip:  5
==============================================================
  L        0.000314      0.500482      0.054146      0.500490
  L        0.054146      0.500490      0.129800      0.500523
  L        0.129800      0.500523      0.180690      0.500562
  L        0.180690      0.500562      0.255261      0.500646
  L        0.255261      0.500646      0.305975      0.500723
 ...
```

The value $NSkip = 5$ was chosen because sequences of line vectors to approximate a smooth curve should extend over several elements. Copy the list to the clipboard.

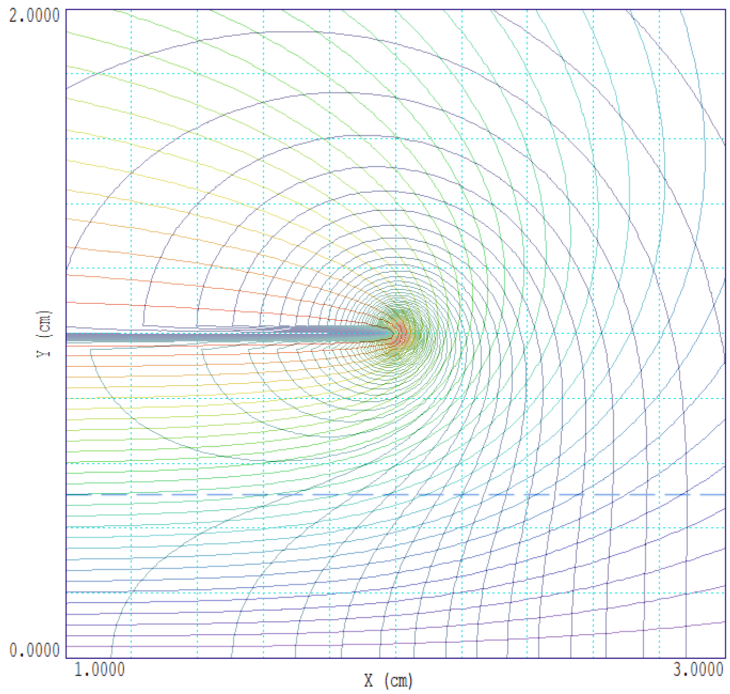

Figure 5: Solution RGInit.EOU, lines of constant ϕ and $|\mathbf{E}|$ near the electrode edge.

The next step is to build a solution with the shaped electrode. In the **Mesh** *Drawing editor*, follow the same initial steps as previously. Create a solution space with dimensions $0.0 \leq x, y \leq 5.0$ and define Region 1 as a rectangle that fills the solution volume. Click the *Start next region* button to enter Region 2. To create dummy content, draw a single Line from (0.0,1.0) to (1.0,1.0). Then click *Start next region* to enter Region 3, representing the fixed ground potential. Draw lines along the bottom, top and right boundaries. Click *Settings/Region properties* and name the three regions AIR, ELECTRODE and GROUND. Set the *Filled* property for the AIR region. Click *Export MIN file* and save the template as RG.MIN. Return to the main menu.

Click the *File/Edit file* button and open *RG.MIN*. Paste the contents of the clipboard to replace the single line in Region 2:

```
REGION   ELECTRODE
  L       0.000000     1.000000     1.000000     1.000000
END
```

The region boundary is now defined by 93 line vectors. Save the file and exit the text editor. Click *Load script (MIN)* to load the modified RG.MIN and click *Edit mesh (Graphics)*. Figure 6*a* shows a magnified view of the display. The electrode region consists of an open set of short line vectors. We can use operations in the **Mesh** *Drawing editor* to 1) modify the shape in the low field region so that it looks like a laser electrode and 2) self-connect the vector set to define a *Filled* region. Use the *Select window* and *Delete selection* operations to remove the top of the shape (Fig. 6*b*).

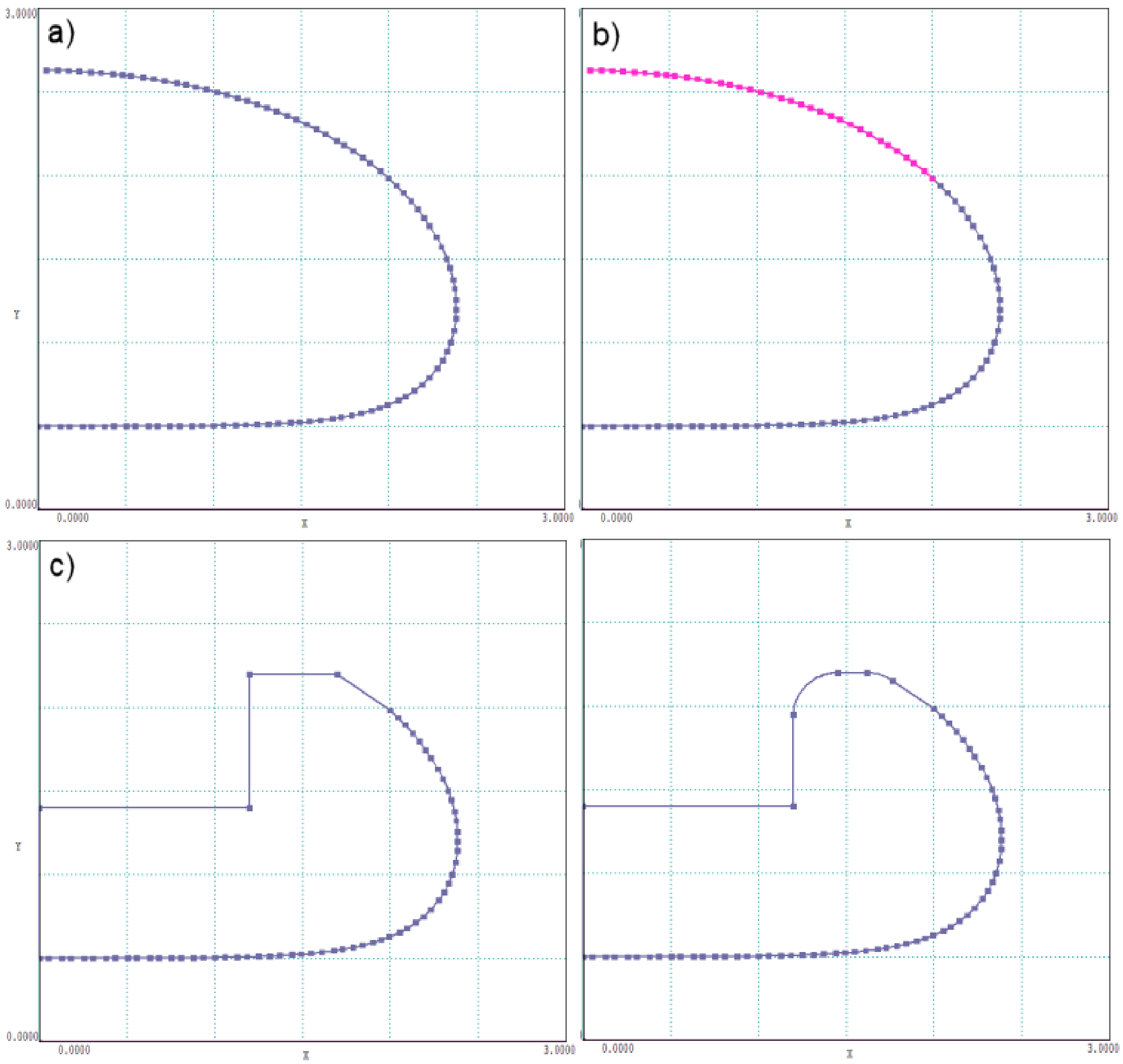

Figure 6: Operations to modify the electrode, changing the shape in the low field region and connecting the vectors to represent a `FILLED` structure.

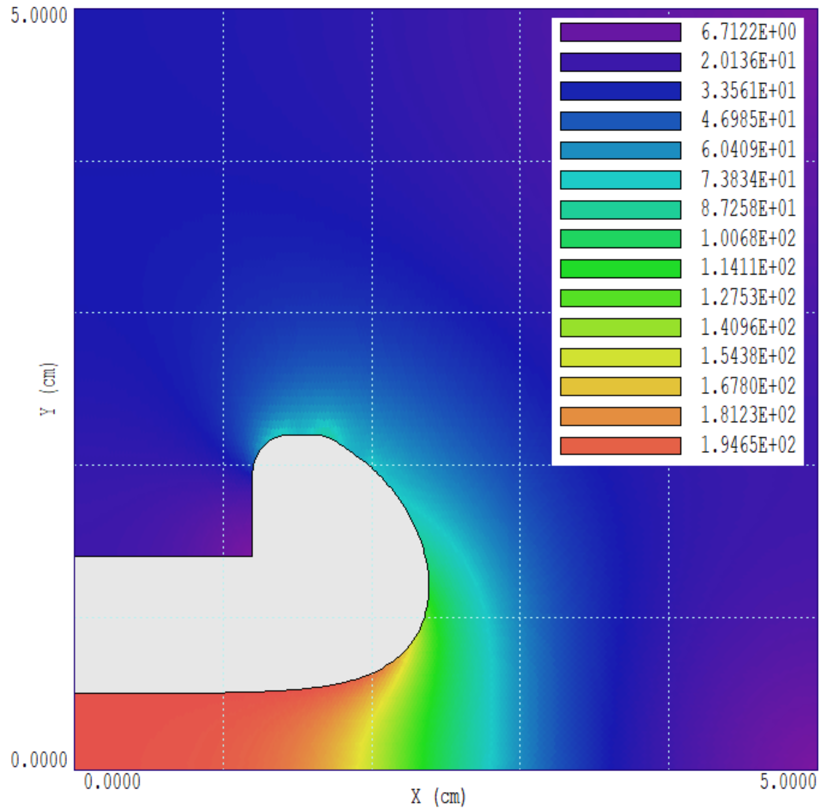

Figure 7: Plot of $|\mathbf{E}|$ in the solution with a modified Rogowski profile electrode.

Start a line vector, then press *F2* and choose *Endpoint snap*. Click on the end of the last shape vector to define the start point. Return to *Grid snap* and use multiple line vectors to define a shape like Fig. 6*c*. Use Endpoint snap to connect the last line to the start of the first shape vector. Use fillet operations to remove sharp corners (Fig. 6*d*). Click on *Region properties* and set Region 2 as *Filled*. The save the work as `RG.MIN`. Exit the drawing editor, then load, process and save the modified mesh script. Run **EStat** and follow actions similar to those for the `RGInit` solution to create an input script `RG.EIN`. Figure 7 shows a plot of $|\mathbf{E}|$ in the solution. There is a broad region with field value $|\mathbf{E}| = 200$ V/m. The field magnitude decreases moving away from center, both on the electrode surface and a points inside.

We can now turn to a solution where the numerical approach is essential: a cylindrical geometry with a more complex generating shape. We will model a high-voltage electrode in a coaxial system that generates a circular region of uniform electric field on a grounded plate. The method is almost identical to that used for the Rogowski profile, so we will summarize operations. The upper section of Fig. 8 shows an r-z plot of the solution geometry defined by `CylinInit.MIN`. The vacuum solution volume (Region 1) has radius 5.0 and length 5.0. The left and outer boundaries are grounded (Region 3). The generating electrode (Region 2) consists of two lines: one from r = 0.0 to 2.0 at z = 1.0 and the other along the axis from z = 1.0 to 5.0. Again, variable resolution is introduced

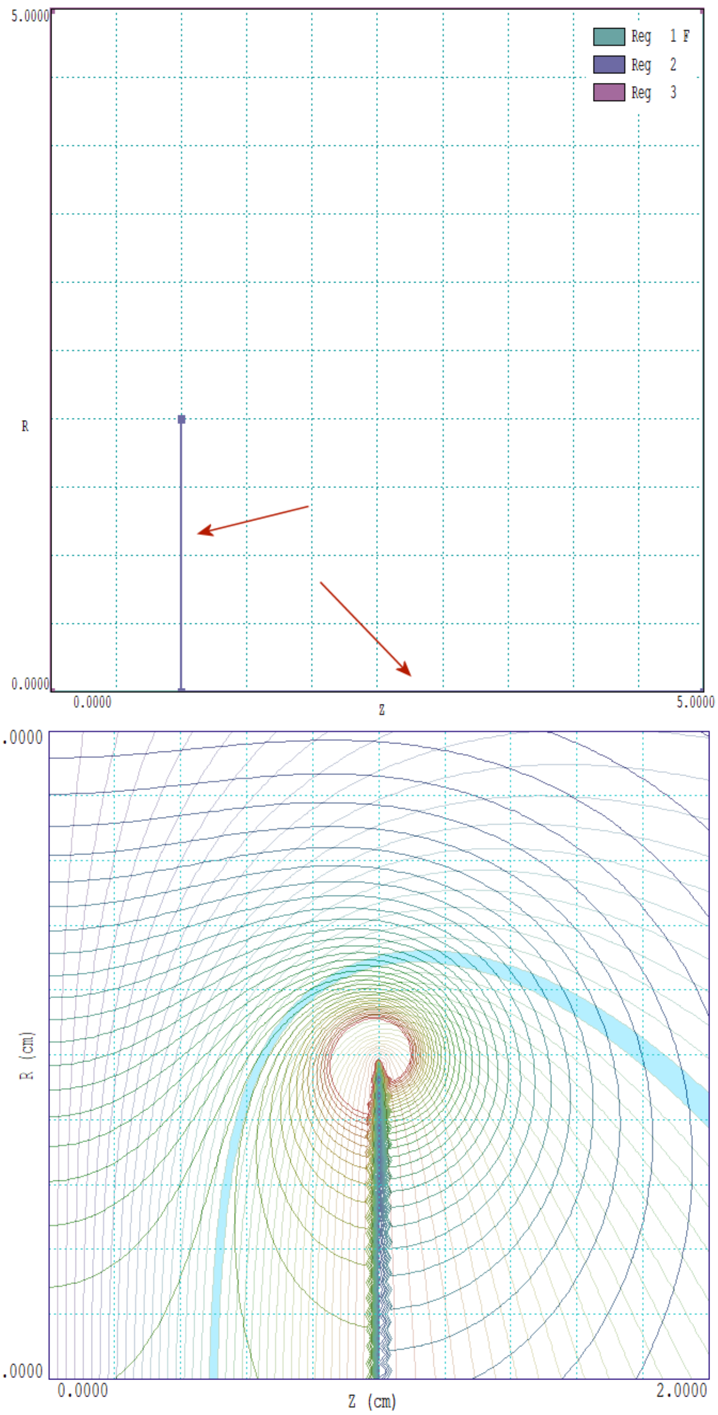

Figure 8: The solution CylinInit. Upper: geometry of the cylindrical system, grounded walls on the left and top boundaries. The generating electrode consists of the designated lines. Lower: superimposed lines ϕ and $|\mathbf{E}|$ near the other electrode edge.

in the foundation mesh. Note that the geometry of the generating electrodes need not consist of lines or elements at the same potential. They may include arcs, points or filled structures, giving the method considerable versatility.

The lower section of Fig. 8 shows superimposed contour plots of ϕ and $|\mathbf{E}|$ near the edge of the radial electrode in the electrostatic solution with an electrode potential 1.0 V. As before, we seek an equipotential line where the electric field magnitude decreases moving away from center. The blue shading indicates the choice that yields the largest volume of uniform field, the equipotential line $\phi = 0.498$ V. The resulting equipotential line was introduced with the text editor into a working solution, `Cylin.MIN`. The upper section of Fig. 9 shows the original set of equipotential vectors defining the electrode. The lower section shows the edited version of the electrode in the **Mesh** *Drawing editor*, a more practical shape that connects to form a filled region. The **Mesh** input file can be exported from the drawing editor in DXF format for use in CAD programs or computer controlled machine tools. Finally, Fig. 10 shows the field magnitude for the electrostatic solution. Again, the method gives an extended region of uniform electric field with an amplitude that decreases moving out along the electrode surface.

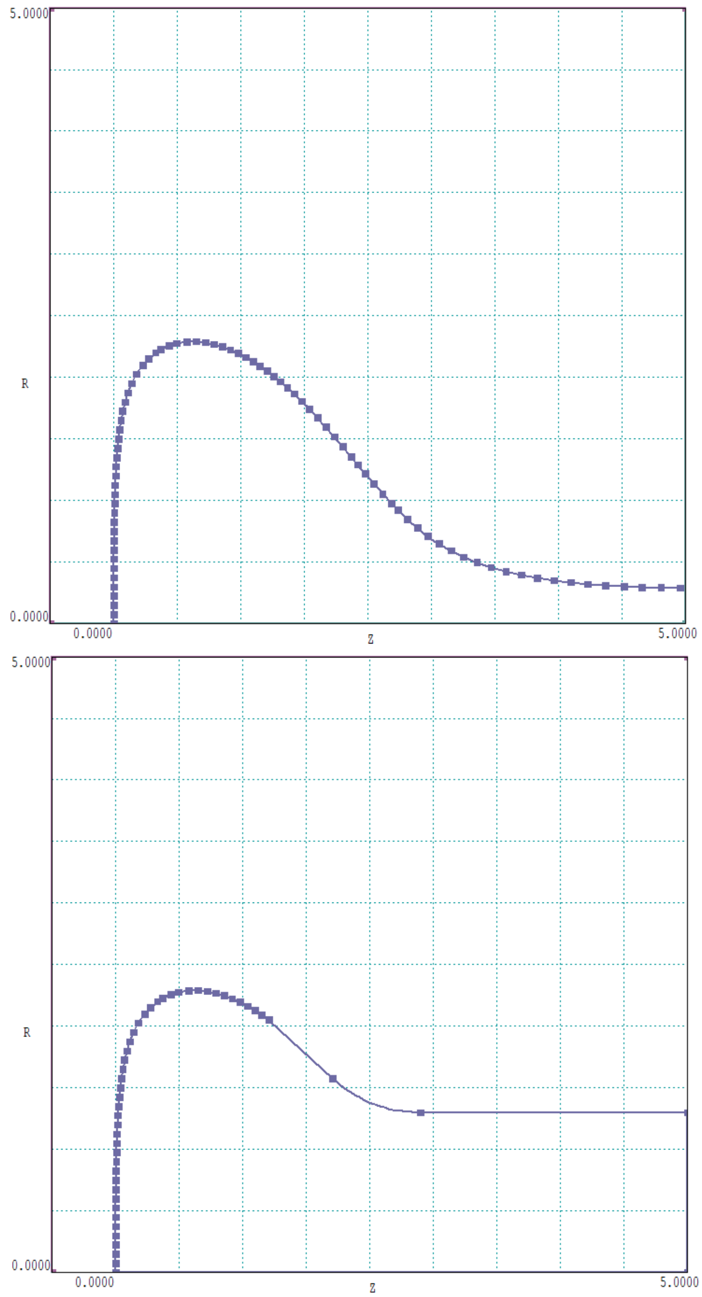

Figure 9: The solution `CYLIN.MIN` viewed in the **Mesh** *Drawing editor*. Upper: electrode defined as an open region by the equipotential line $\phi = 0.498$ V. Lower: Edited electrode with connected vectors to defined a filled region.

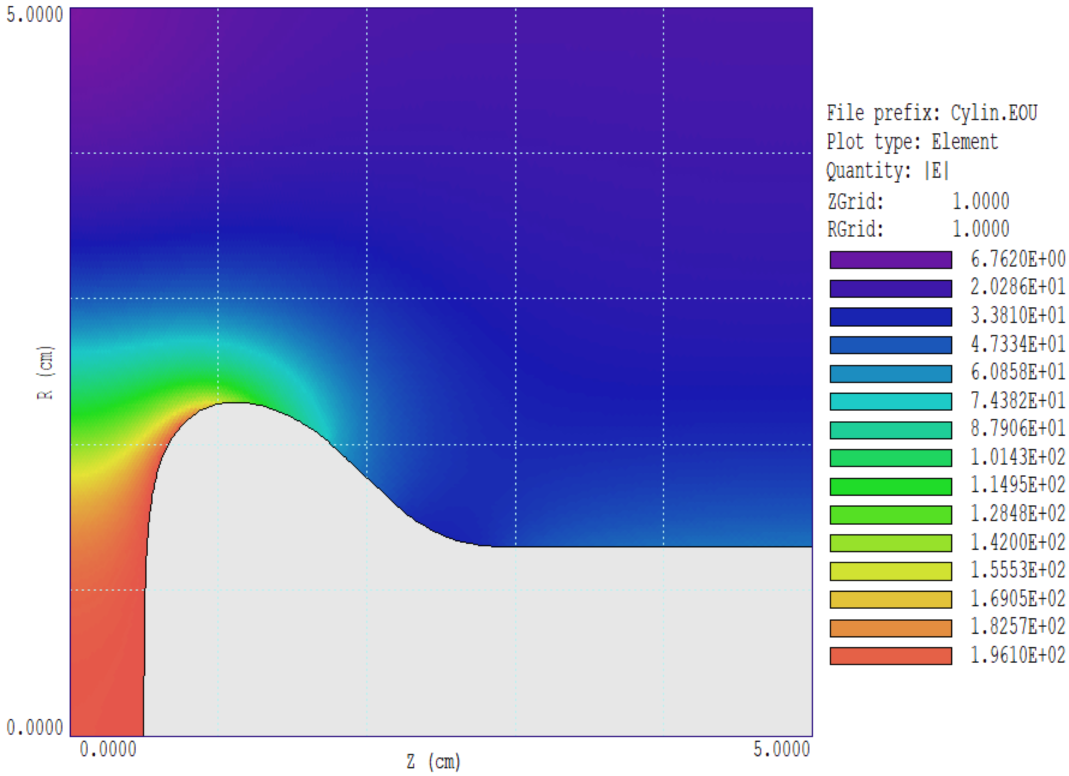

Figure 10: Electrostatic solution `CYLIN`, plot of $|\mathbf{E}|$.