



Modeling a cylindrical charged-particle beam deflector

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This tutorial discusses a method to create a uniform transverse field in a curved electrostatic beam transport system. It addresses two-dimensional models of systems that approximate fields of the form $\mathbf{E} = E_0\mathbf{x}$ in a cylindrical transport tube of radius R_0 with infinite length in z . A subsequent tutorial addresses a practical curved transport system for macroparticles, a periodic array of einzel lenses and deflectors. A second function of the report is to illustrate techniques in the **Mesh** program to create conformal meshes of systems with multiple symmetric objects.

As shown in Figure 1, a uniform field of magnitude V_0/R_0 could be created using infinite parallel plates with voltages $+V_0$ and $-V_0$ at positions $x = -R_0$ and $x = R_0$. The figure includes a circle of radius R_0 with center midway between the plates. The potential at positions on the circle is given by

$$\phi = V_0 \cos(\theta) \tag{1}$$

Suppose we placed a large set of electrodes on the circle with applied potentials that follow Eq. 1. The internal field would be unchanged and we could remove the parallel plates, eliminating most of the external field. We will use the **Electrostatics Toolkit** to investigate configurations with a finite number of electrodes and see how closely the system approximates the ideal variation of Eq. 1. If there are N electrodes and the electrode at $x = R_0$ is designated as number $n = 0$, then the applied potentials are given by

$$\phi_n = V_0 \cos(2\pi n/N), \tag{2}$$

for $n = 0$ to $n = N - 1$.

In the calculations, the electrodes are wedge sections with inner radius $R_0 = 1.0$ cm and outer radius 1.2 cm. Calculations were performed for $N = 2, 4, 6, 8, 10$ and 12. The electrodes have average azimuthal positions consistent with Eq. 2 and are separated by small gaps. We will discuss generating input for $N = 12$ using the **Mesh Drawing Editor**. Run the program and click on the tool **New mesh (graphics)**. Figure 2a shows the opening dialog with the run parameters. The first activity is to define the solution volume (Region 1), the circular cross section of a cylindrical volume with radius 1.2 cm. By default, the drawing editor starts in grid-snap mode and expects input for the first region. Choose the command **Insert/Add circle/Center-point** and click on the origin (0.0, 0.0) and the point (1.2, 0.0) with the mouse. The program generates four arc vectors to outline the region (Fig. 2b).

The next step is to create the first electrode centered at $\theta = 0.0^\circ$. Click the tool **Start next region** to open Region 2 for input. We will define the inner and outer electrode edges as arcs. Click on the command **Insert/Add arc(s)/Start-Center-Angle**. Use the mouse to specify the *Start* point as (1.0,0.0) and the *Center* point as (0.0,0.0). Then enter 14° in the dialog. The full electrode will span 28.0° rather than 30.0° to leave gaps between the full electrode set. Continue with the next arc: *Start* point (1.0,0.0), *Center* point (0.0, 0.0) and angle -14.0° . Then define the outer arcs with *Start* points (1.2,0.0). Figure 2c) shows the configuration. To add the radial lines, pick **Settings/Snap control** and choose *Endpoint*. Then click on **Insert/Add line(s)** and create lines between the arc endpoints on the edges. Save the configuration as **EDeflect12.MIN**.

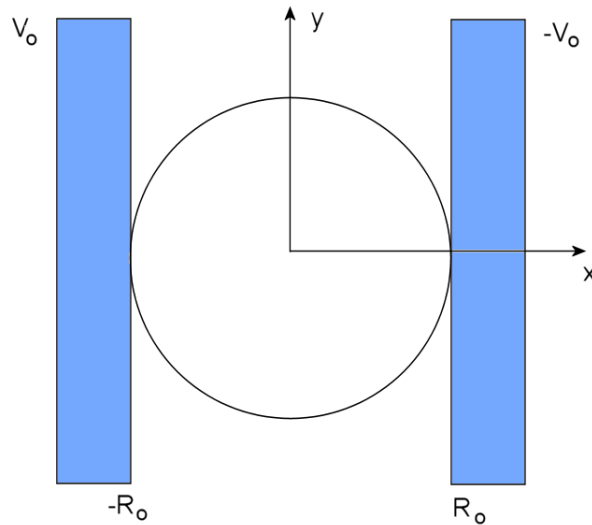


Figure 1: Replacing parallel plates with a potential variation defined on a cylinder to generate a uniform field $E_x = V_0/R_0$.

At this point, there are two ways to define the other 11 electrodes:

- Continue in the **Mesh Drawing Editor**.
- Use *Copy/Paste* commands in a text editor.

We will first run through the additional steps in the *Drawing Editor*. Figure 3 illustrates the procedure. These operations create a second electrode in a new region:

- Click **Insert/Start next region** to open Region 3 for input.
- Click **Edit/Select/Selection region** and mouse-click on a point near the inner edge of the initial electrode.
- Click **Edit/Move selection to current region**.
- Click **Rotate selection** and enter 30.0° in the dialog.

Figure 3a shows the resulting state of the drawing. Click **Deselect all**. Follow the same step sequence to place the additional 10 electrodes in new regions (Fig. 3b). When complete, click **Settings/Region properties** to open a dialog. Check the boxes in the Filled column for Regions 1 through 13. Change the titles to description names like **ELECTRODE01**, **ELECTRODE02**, Finally, save the MIN file.

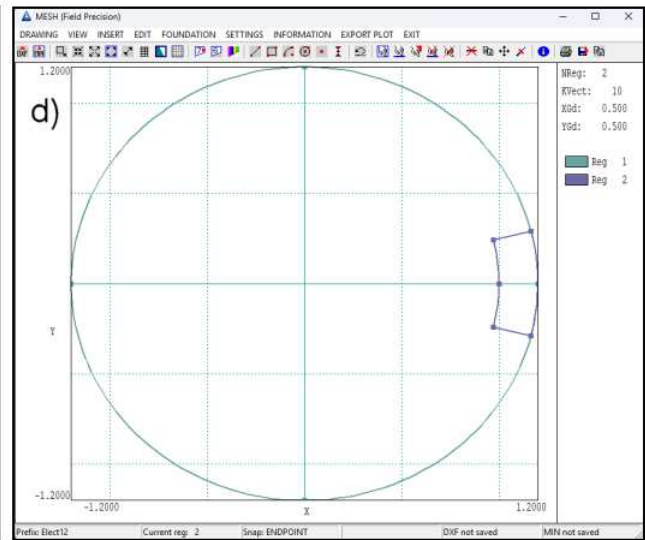
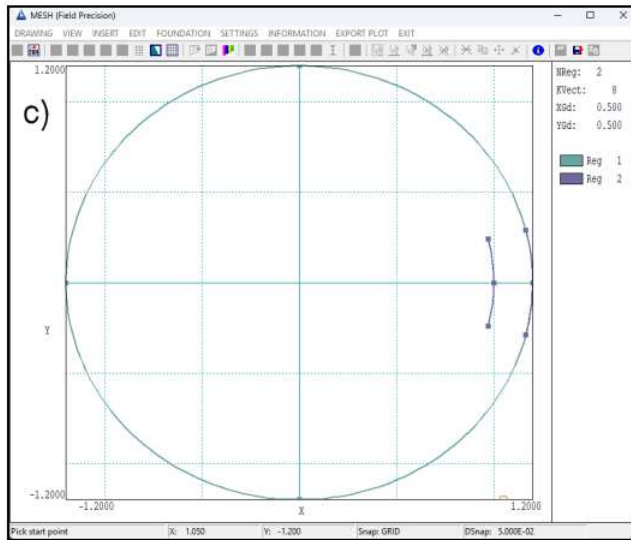
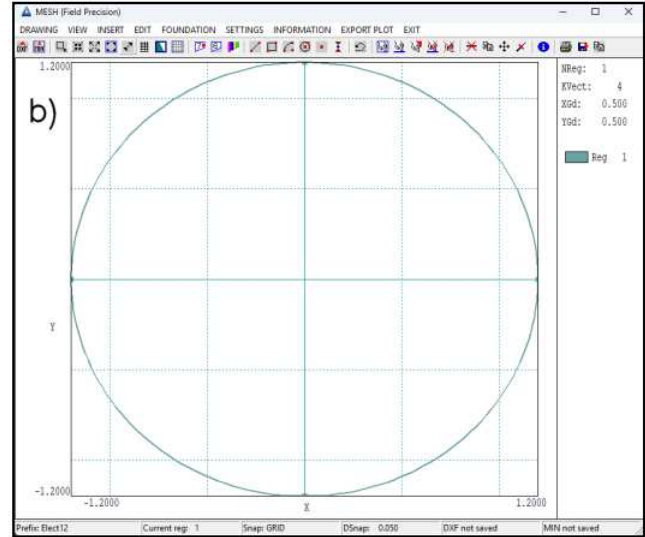
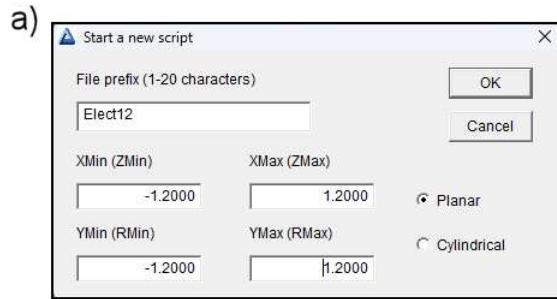


Figure 2: Creating the solution volume and the first of 12 wedge electrodes.

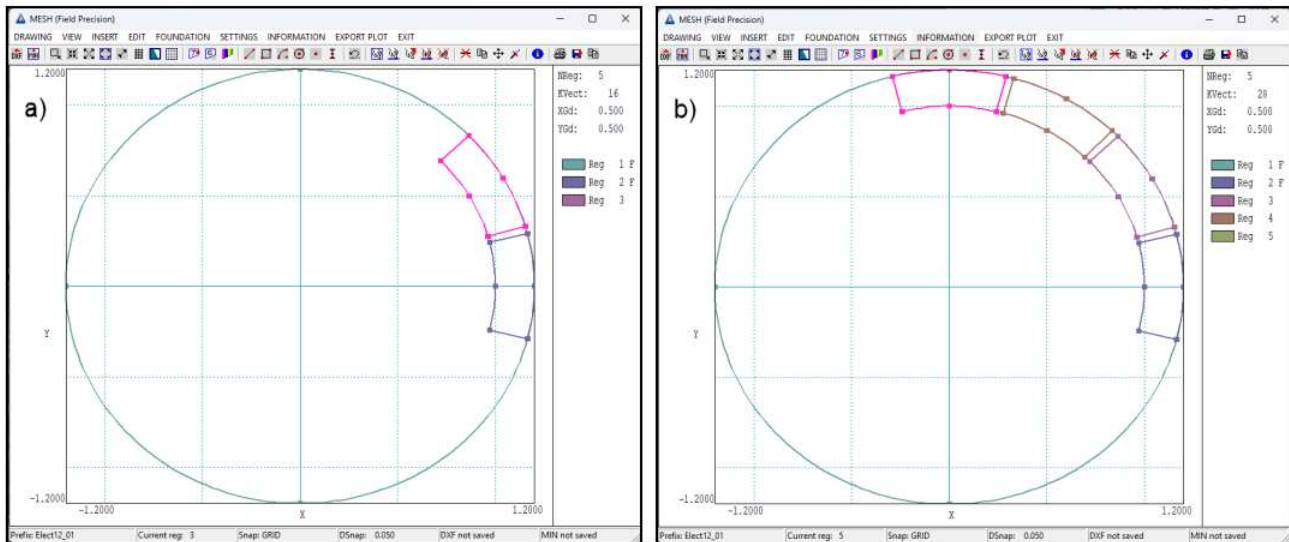


Figure 3: Selecting and duplicating an electrode in a new region, and then applying a rotation to the selection.

An alternative that may save time is to use a text editor. The file `EDeflect12.MIN` looks like this after defining the first electrode:

```
GLOBAL
  XMESH
    -1.200000    1.200000    0.020000
  END
  YMESH
    -1.200000    1.200000    0.020000
  END
END
* -----
REGION REGION001
  A   -1.200000    0.000000    0.000000   -1.200000    0.000000    0.000000
  A    0.000000   -1.200000    1.200000    0.000000    0.000000    0.000000
  A    1.200000    0.000000    0.000000    1.200000    0.000000    0.000000
  A    0.000000    1.200000   -1.200000    0.000000    0.000000    0.000000
END
* -----
REGION REGION002
  A    0.970296    0.241922    1.000000    0.000000    0.000000    0.000000
  A    1.000000    0.000000    0.970296   -0.241922    0.000000    0.000000
  L    0.970296   -0.241922    1.164355   -0.290306
  A    1.164355   -0.290306    1.200000    0.000000    0.000000    0.000000
  A    1.200000    0.000000    1.164355    0.290306    0.000000    0.000000
  L    1.164355    0.290306    0.970296    0.241922
END
* -----
ENDFILE
```

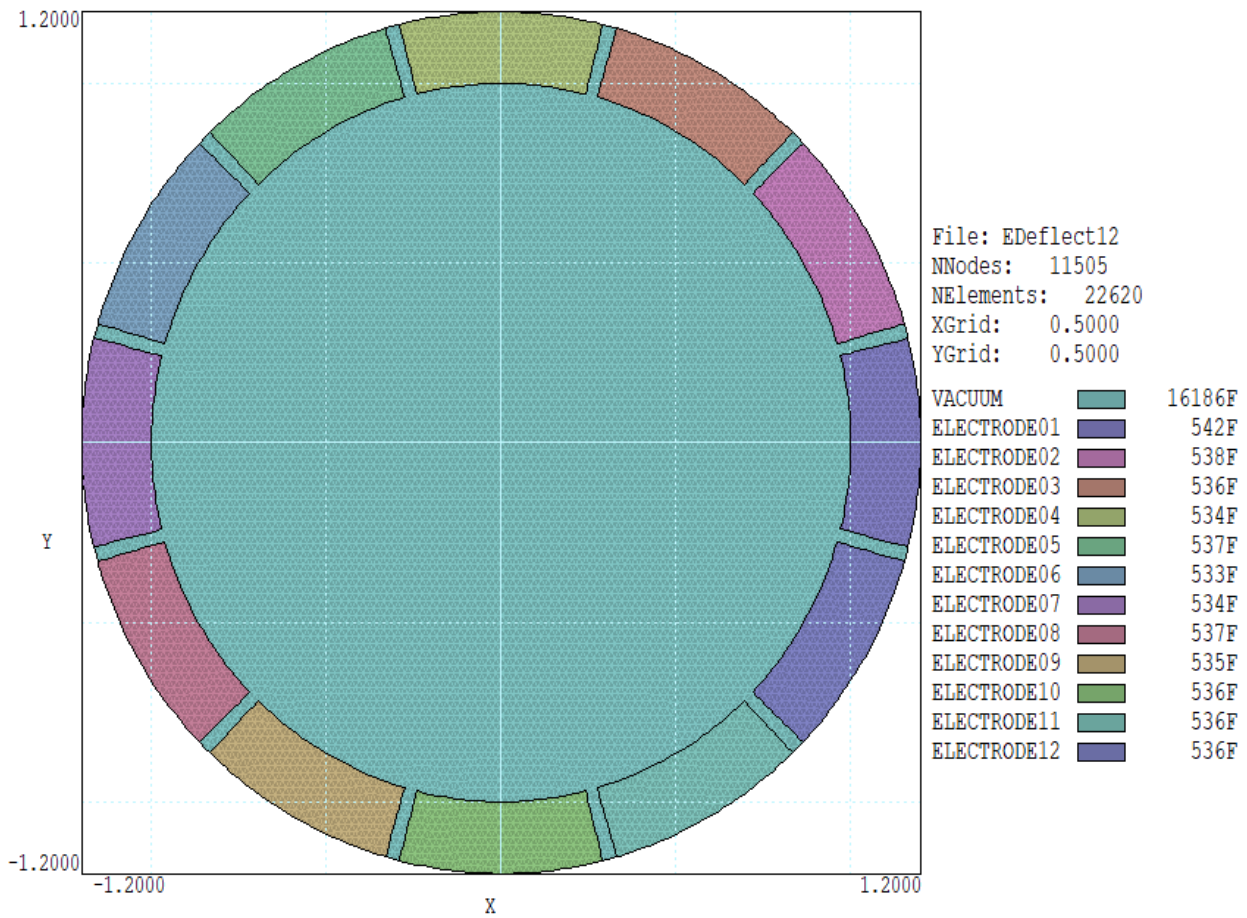


Figure 4: Completed mesh for 12 electrodes.

Make the following revisions:

- Change `REGION REGION001` to `REGION FILL VACUUM`
- Change `REGION REGION002` to `REGION FILL ELECT01`
- Add the command `ROTATE 0.0` to Region 2 above the vector list.

Copy Region 2 and paste it beneath 11 times. With the text editor, change the new regions titles to `ELECT02`, `ELECT03`,.... In the `ROTATE` commands, change the values to `30.0`, `60.0`,..., `330.0`. Save the modified file. Processing the script gives the completed mesh shown in Fig. 4 for an average element dimension of 0.02 cm.

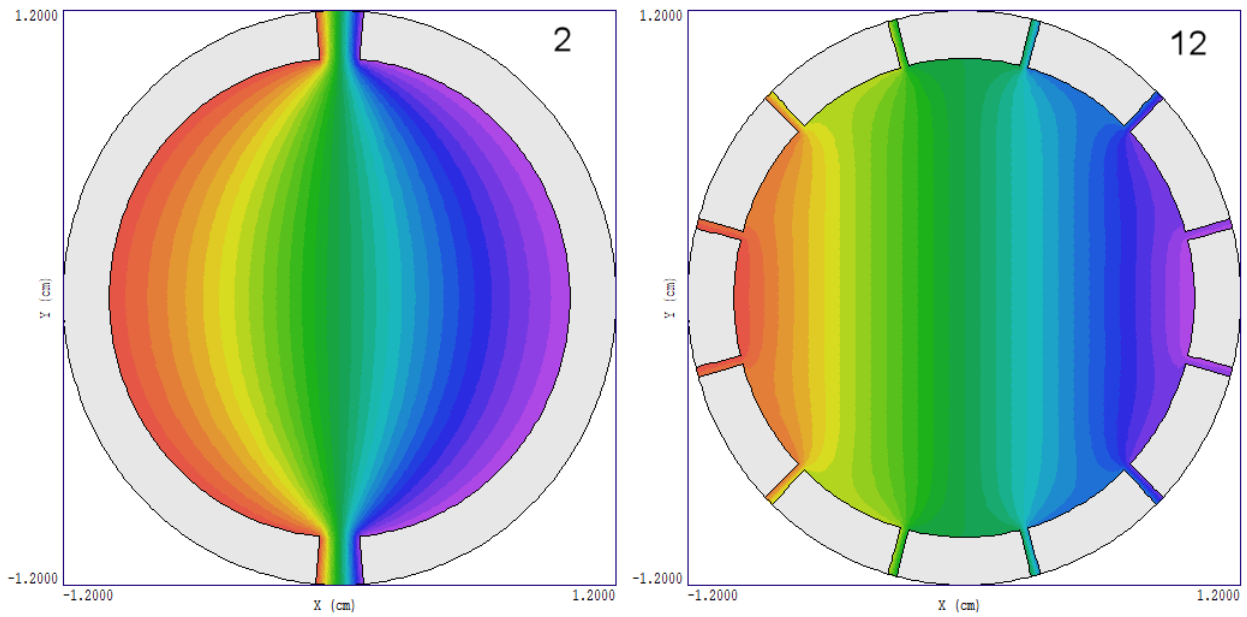


Figure 5: Equipotential contours for $N = 2$ and 12.

Similar operations are used to create **Mesh** input files for the other choices of N . Input scripts for the field calculation can be created in the *Setup* solution dialog of **EStat** using voltage values from Eq. 2. This is the script content for $N = 6$:

```

Mesh = EDeflect06
Geometry = Rect
DUnit = 1.0000E+02
ResTarget = 5.0000E-08
MaxCycle = 5000
* Region 1: VACUUM
Epsi(1) = 1.0000E+00
* Region 2: ELECTRODE01
Potential(2) = -1.0000E+00
* Region 3: ELECTRODE02
Potential(3) = -5.0000E-01
* Region 4: ELECTRODE03
Potential(4) = 5.0000E-01
* Region 5: ELECTRODE04
Potential(5) = 1.0000E+00
* Region 6: ELECTRODE05
Potential(6) = 5.0000E-01
* Region 7: ELECTRODE06
Potential(7) = -5.0000E-01
EndFile

```

Figure 5 shows equipotential contours for a small number ($N = 2$) and large number ($N = 12$) of electrodes. In the two-electrode solution, the contour boundaries are curved and the contour width (inversely proportional to the electric field) varies considerably over the solution volume. In contrast, the contour boundaries are largely parallel and the widths are almost uniform for $N = 12$. We can generate quantitative information by plotting the variation of the deflecting field magnitude, $|E_x|$ over the solution volume. The ideal field with a large number of electrodes is $E_x = 100.0$ V/m. Figure 6 shows contours for six different values of N over the range $E_x = 90.0$ V/m to 110.0 V/m. The solution with $N = 12$ using seven voltage levels (-1.000, -0.866, -0.500, 0.000, 0.500, 0.866 and 1.000 V) gives a central region of good deflection field with diameter approximately 1.3 cm.

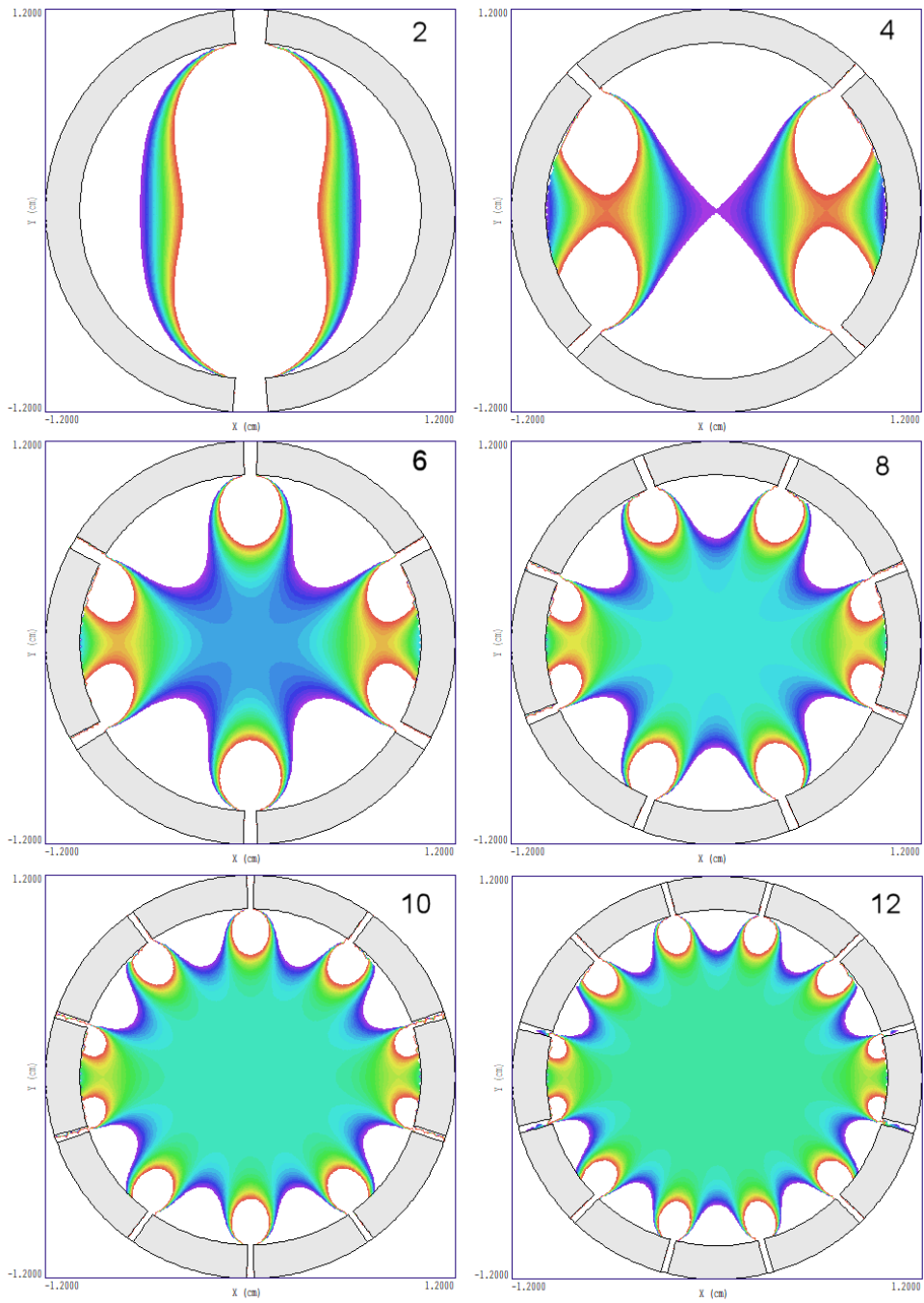


Figure 6: Spatial variation of E_x , color contours over the range 90.0 V/m (violet) to 110.0 V/m (red).