A common issue for X-ray radiography is the resolution of small details. Analytic and semi-analytic calculations can give useful results, but often entail simplifying approximations. A full Monte Carlo imaging simulation is computationally intensive and limited by statistical variations. On the other hand, the Monte Carlo approach includes the full effects of the photon energy distributions and scattering. This tutorial illustrates how GamBet may be used to find spatial variations of transmitted X-ray flux in the presence of small imperfections. The calculations make extensive use GenDist and other available tools in the GamBet package.

We shall study propagation of photons in the energy range 600 keV through a steel target. The photons are generated by an ideal point source at an infinite distance. We use a planar two-dimensional geometry to achieve good statistics with moderate run time. The NIST XCom site gives a total attenuation constant for 600 keV photons in iron of $\mu = 0.61 \text{ cm}^{-1}$. The scale length for significant photon absorption is on the order of $1/\mu = 1.65 \text{ cm}$.

In the calculations, photons move in the $+x$ direction and enter at $x = 0.0^+ \text{ cm}$. In the geometry mesh, Region 1 is iron with thickness along $x$ of 3.0 cm and Region 2 is a drift space (Void) of length 15.0 cm. The detector is located at the exit plane, $x = 18.0 \text{ cm}$. The photon beam has uniform density in $y$ with a width of 2.0 cm. The first step is to determine the spatial spreading of the photon flux in the detector plane resulting from scattering events. Run RADIMAGE01 has a single primary photon of energy 600 keV entering a $y = 0.0$. The parameter $NPMult$ is set so that the run generates 100,000 showers. Figure 1 shows the resulting distribution of transmitted photons in the plane $x = 18.0 \text{ cm}$ (bin width equals 0.4 mm). The distribution of scattered photons is approximately uniform in $y$. The scattered flux is a fraction $f_s = 3 \times 10^{-3}$ of the primary flux. From this figure, we can estimate a limit on resolution set by photon scattering. Suppose we have a uniform photon beam of width $W$ incident on the plate and we want to resolve a groove of width $\delta$. The fraction of scattered photons over the width of the imperfection is approximately $F = W f_s/\delta$. For an imperfection of height 1.0 mm in the 3.0 cm steel plate, the value is $F = 0.06$. The groove will be clearly visible if it results in a fractional change of the detector-plane flux equal to or greater than this value.

The series of runs RADIMAGE02, RADIMAGE03 and RADIMAGE04 addresses resolution limits for illumination with monoenergetic photons. The incident beam has kinetic energy 600 keV with uniform density over a width $W = 2.0 \text{ cm}$ in $y$. As shown in Fig. 2, we introduce a small groove with $\delta = 1.0 \text{ mm}$ and different depths (5.0, 2.0 and 1.0 mm) and measure the resulting photon flux at the detector plane. It is first necessary to create a distribution of primary photons distributed uniformly in $y$ between $\pm 1.0 \text{ cm}$. We use GenDist with
In response to the script, the program creates the file RADIMAGE.SRC with 20,000 particles evenly spaced along $y$ with direction vectors pointing in $x$.

With the parameter value $NP\text{Mult} = 25$, each GamBet calculation includes 500,000 showers and takes about 18 minutes on a 3.2 GHz computer. The escape file contains about 360,000 photons. The number drops to 46,000 when a filter is applied that limits photons to the exit plane in the range $-0.5 \leq y \leq 0.5$. Figure 3 shows histograms of photon flux in the detector plane with bin width 1.0 mm (equal to the width of the groove) for depths of 5.0 mm, 2.0 mm and 1.0 mm. In all cases, the object is clearly visible over the statistical variations of background flux The statistical variations of $\sim \pm 1.6\%$ are consistent with the number of photons per bin, $N_b \approx 4200$. The predicted standard deviation is
Figure 2: Mesh for run RADIMAGE03. Region 3 represents a groove of width 1.0 mm and depth 2.0 mm on the exit face of the steel plate.

<table>
<thead>
<tr>
<th>Pit depth</th>
<th>Attenuation</th>
<th>Prediction</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>0.2198</td>
<td>1.354</td>
<td>1.314</td>
</tr>
<tr>
<td>2.0</td>
<td>0.1832</td>
<td>1.129</td>
<td>1.140</td>
</tr>
<tr>
<td>1.0</td>
<td>0.1725</td>
<td>1.063</td>
<td>1.059</td>
</tr>
<tr>
<td>0.0</td>
<td>0.1623</td>
<td>1.000</td>
<td></td>
</tr>
</tbody>
</table>

\[ \sigma \approx \frac{1}{\sqrt{N_b}} = 0.0154 \] (1)

Table 1 summarizes the results. The quantity in Column 2 is the attenuation factor, \( \exp(-\mu D) \). Here, \( D \) is the depth of the plate with the inclusion of the groove. Column 3 shows the predicted signal enhancement as a function of pit depth, determined by taking ratios of quantities in Column 2. Finally, Column 4 shows the code prediction based on the analysis of the results illustrated in Fig. 3.

As a final task, we shall inject a realistic photon energy spectrum to image a groove of depth 2.0 mm. The photons are created by bremsstrahlung interactions of a 1.5 MeV electron beam incident on a tungsten target of thickness 1.4 mm. An initial calculation (SPECTRUM) is performed using a single primary electron with \( NPMult = 10000 \) and a bremsstrahlung forcing factor 50.0. Figure 4a shows resulting energy-weighted photon flux emerging from the target in the forward direction. As expected, the distribution approximates a straight-line curve at high energy. The low-energy cutoff is about
Figure 3: Histograms of photon flux in the detector plane as a function of $y$ for grooves of width 1.0 mm and depth 5.0 mm (top), 2.0 mm (middle) and 1.0 mm (bottom). Bin width: 1.0 mm
0.3 MeV.

For an imaging simulation, we want uniformly distributed photons along \( y \) to form a beam. At all points, the displacement of photon energy from the central value should be assigned with weighting to approximate the spectrum of Fig. 4a. One possible approach is to approximate the spectrum by an analytic function, create a table of values using the *Table generator* tool of **GenDist** and then to use the program to create a **SRC** file of incident photons for **GamBet**. As an analytic probability function, we shall use a function that rises sharply to 1.0 at 0.3 MeV and the decreases linearly to zero 0.0 at 1.5 MeV. In this case, the central energy of 0.9 MeV with displacement limits of ±0.6 MeV. The following function creates the desired distribution when used in *Table generator* tool of **GenDist**:

\[
 f(x) = 0.25 \cdot (1200000. - ((x+900000) -300000))/(x+900000)
\]

The results are saved as the file **SPECTRUM**. The file is then used as input to **GenDist** to create the desired distribution in \( y \) and \( T \). The file **SPECTRUM.DST** has the following content:

```
* GenDist 4.0 Script (Field Precision)
* File: SPECTRUM.DST
FileType = SRC
Particle = P
Energy = 9.0000E+05
Def(Rect) = 0.0000E+00 1.0000E+00 1 20000
Rotate = 0.0000E+00 9.0000E+01 0.0000E+00
Distribution = Random
TDist = Spectrum
EndFile
```

Note the use of random weighting. With uniform weighting, the energy displacement would be correlated with position in \( y \). The end result is the file **SPECTRUM.SRC** used as input to **RADIMAGE05**.

Figure 4b shows the resulting energy flux spectrum of injected photons. Finally, Fig. 5 shows the photon flux distribution in the detector plane after passing through a 3.0 cm steel plate with a groove of width 1.0 mm and depth 2.0 mm. The relative change of photon energy flux is comparable the result of Fig. 3(middle). Statistical variations between bins are somewhat higher with the impressed energy distribution.
Figure 4: Spectrum of forward-directed bremsstrahlung photons produces by a 1.5 MeV electron beam incident on a 1.4 mm thick tungsten target. Top: GamBet result. Bottom: distribution of electrons injected in RADIMAGE05 from a mathematical approximation.
Figure 5: Histogram of photon energy flux in the detector plane as a function of $y$ for a groove of width 1.0 mm and depth 2.0 mm. The energy of incident photons follows an approximate bremsstrahlung distribution. Bin width: 1.0 mm.