Tutorial: Representing cathode temperature effects in Trak and OmniTrak

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This tutorial reviews theoretical considerations for including the effects of cathode temperature in simulations of high-current, steady-state electron guns with ray-tracing codes such as **Trak** and **OmniTrak**. The text S. Humphries, **Charged Particle Beams**, available for download at:

`http://www.fieldp.com/cpb.html`

reviews the relevant beam physics. Thermal distributions of transverse energy limit the performance of cylindrical guns with high compression ratio for generating small-diameter beams with high current density. Injection emittance also introduces limits in the performance of planar guns to generate narrow sheet beams for high-power microwave generation. To review, electron guns may operate in either the *source-limited* or *space-charge-limited* mode. In the first case, the source current density j_s is given by the Richardson-Dushman law. Here, the current density depends on the cathode material properties and temperature. The space-charge-limited current density j_c is determined by the Child law which involves the properties of the extraction gap. A gun is space-charge limited when $j_s > j_c$. There are two reasons to operate in this mode:

- **Spatial uniformity**: the quantity j_s may vary substantially if there are differences in the cathode surface work function or temperature.
- **Temporal uniformity**: the gun current will remain constant over time even if there are changes in the cathode properties.

In normal operation, the cathode temperature in electron guns for high-quality beams is sufficiently high so that $j_s > j_c$ over the emission surface.

To begin, consider the case where $j_s = j_c$. Here, every electron generated at the cathode surface is extracted. The defining condition for Child-law emission is that the beam space-charge reduces the electric field on the cathode surface to zero (Fig. 1a). In a ray-tracing code, it is not possible to initiate computational electrons at the physical cathode surface because they would never leave their start point. The standard approach is to initiate model electrons from an emission surface a short distance from the cathode where the electric field is non-zero. The kinetic energy of the computational electrons and the current density that they represent are determined from analytic expressions. In the absence of cathode temperature effects, the model electrons leave the emission surface parallel to the direction of the local electric field.

To represent the role of cathode temperature in the transverse electron dynamics of a space-charge limited gun, one approach is to add small angular displacements to model particle orbits at the emission surface. The

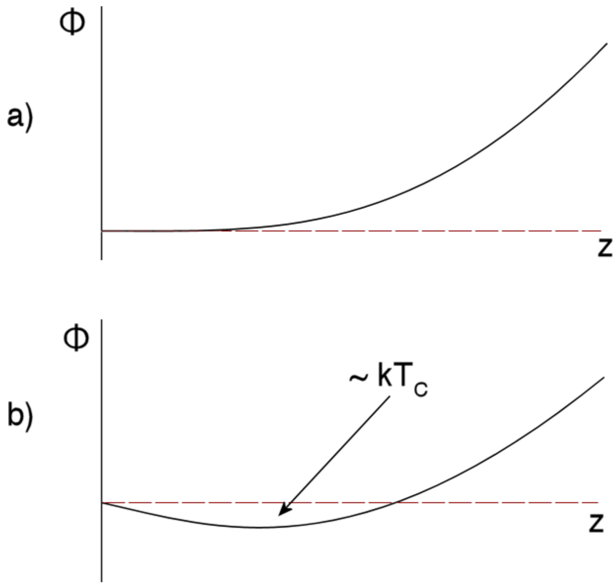

Figure 1: Variation of electrostatic potential near the surface of a thermionic emitter. *a*) Source flux equals the Child-law flux. *b*) Source flux exceeds the Child-law flux, resulting in selection of electrons with higher longitudinal energy and lower transverse energy.

following method is used in the **Trak** and **OmniTrak** codes. The probability distribution of transverse velocity $v_\perp$ of particles emitted from a source at temperature T_s approximates the Maxwell distribution:

$$f(v_\perp)dv_\perp = v_\perp \left(\frac{m}{kT_s}\right) \exp\left(-\frac{mv_\perp^2}{2kT_s}\right). \tag{1}$$

We can rewrite Eq. 1 in terms of the normalized variable

$$\chi = \sqrt{\frac{m}{2kT_s}}\, v_\perp, \tag{2}$$

as

$$f(\chi)d\chi = 2\chi \exp\left(-\chi^2\right)\, d\chi. \tag{3}$$

The integral of Eq. 3 gives the cumulative probability, a variable in the range from 0.0 to 1.0:

$$\int_0^\chi f(\chi)d\chi = \zeta = 1 - \exp(-\chi^2). \tag{4}$$

The procedure in the codes is to generate a random value $0.0 \le \zeta \le 1.00$ and then to find a weighted value of χ from the inverse of Eq. 4:

$$\chi = -\frac{\ln(1-\zeta)}{2}. \tag{5}$$

The angle of the model particle relative to the local electric fieldis given by

$$\Delta\theta \cong \frac{v_\perp}{v_0} = \chi \sqrt{\frac{kT_s}{T_p}}, \tag{6}$$

where T_p is the kinetic energy of the particle at the emission surface. Particles are emitted with a random distribution of azimuthal angle about the surface normal vector. The procedure is valid in the limit that $v_\perp \ll v_0$.

If all electrons created at the cathode are extracted ($j_s = j_c$), we can identify the effective source temperature as the physical temperature of the cathode, $T_s = T_c$. It is important to recognize that T_s may be lower than T_c when $j_s > j_c$. Figure 1*b* illustrates the effect. The electron distribution at the cathode surface is Maxwellian with temperature T_c. When the source flux exceeds the Child-law value, a portion of the incident electrons reflect back to the cathode. The potential exhibits the sheath variation shown in Fig. 1*b*. The depth of the potential barrier ensures that only a fraction of the available electron flux equal to j_c passes. This fraction has the highest longitudinal energy and (on the average) a reduced transverse energy. Under typical operating conditions, the sheath is thin (a few μm) and therefore has a negligible effect on the value of j_c.

We can determine a quantitative relationship between T_s and T_c. Assume that electrons leaving the cathode surface have the three-dimensional velocity distribution,

$$f(v_x, v_y, v_z) = \left(\frac{m_e}{2\pi kT_c}\right)^{3/2} \exp\left[-\frac{m_e(v_x^2+v_y^2+v_z^2)}{2kT_c}\right]. \tag{7}$$

Equation 7 can be rewritten in terms of the dimensionless velocities

$$V_x = \sqrt{\frac{m_e}{2kT_c}}\, v_x,\ V_y = \sqrt{\frac{m_e}{2kT_c}}\, v_y,\ V_z = \sqrt{\frac{m_e}{2kT_c}}\, v_z. \tag{8}$$

as

$$f(V_x, V_y, V_z) = A\ \exp\left[-(V_x^2+V_y^2+V_z^2)\right]. \tag{9}$$

Suppose that the sheath potential barrier allows extraction of only those electrons with $V_z > V_{z0}$. The fraction of the original distribution that passes the sheath is

$$\eta = 1 - \mathrm{erf}(V_{z0}) \tag{10}$$

where the error function is given by

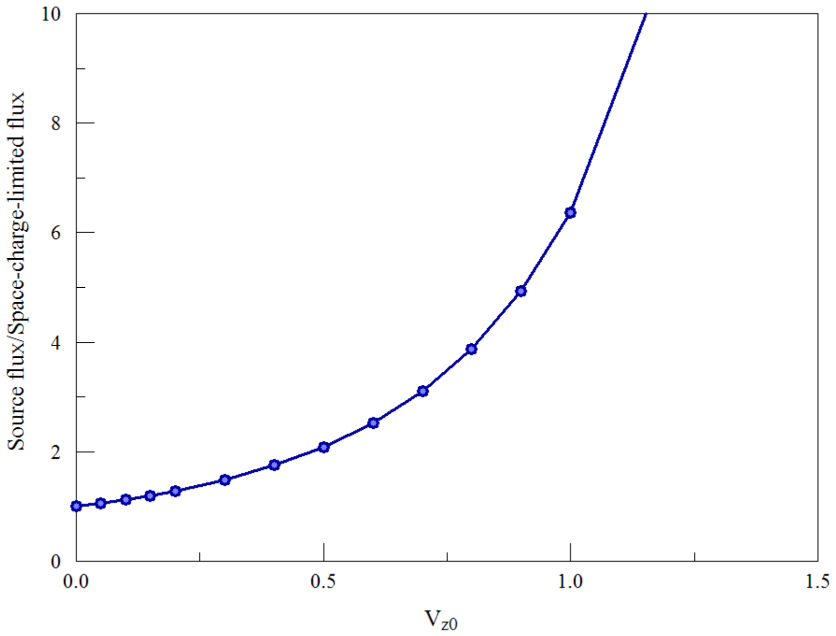

Figure 2: Relationship between (j_s/j_c) and the dimensionless cutoff velocity $V_{z0} = \sqrt{m_e/2kT_c} v_z$.

$$\mathrm{erf}(V_{z0}) = \frac{2}{\pi} \int_0^{V_{z0}} \exp(-t^2) dt. \tag{11}$$

By definition, the quantity $(1/\eta)$ equals the ratio of the source-limited flux to the space-charge-limited value:

$$\frac{1}{\eta} = \frac{j_s}{j_c}. \tag{12}$$

Figure 2 plots the flux ratio $1/\eta$ versus V_{z0} determined from Eq. 10. In a typical system, we might expect an average ratio $j_s/j_c \geq 2$ to ensure that $j_s > j_c$ over the entire surface. This value corresponds to the dimensionless velocity cutoff $V_{z0} \cong 0.5$.

To conclude, we must find how the limit on V_z affects the average transverse velocity of transmitted electrons. One method is to perform a three-dimensional numerical integrations of the expression:

$$\int_0^\infty dV_x \int_0^\infty dV_y \int_0^\infty dV_z \sqrt{V_x^2 + V_y^2} \, \exp\left[-(V_x^2 + V_y^2 + V_z^2)\right] \tag{13}$$

The integrated quantity is the transverse velocity weighted by the probability distribution. The normalization integral extends over the full velocity range. In the second integral, we include contributions only from cells where

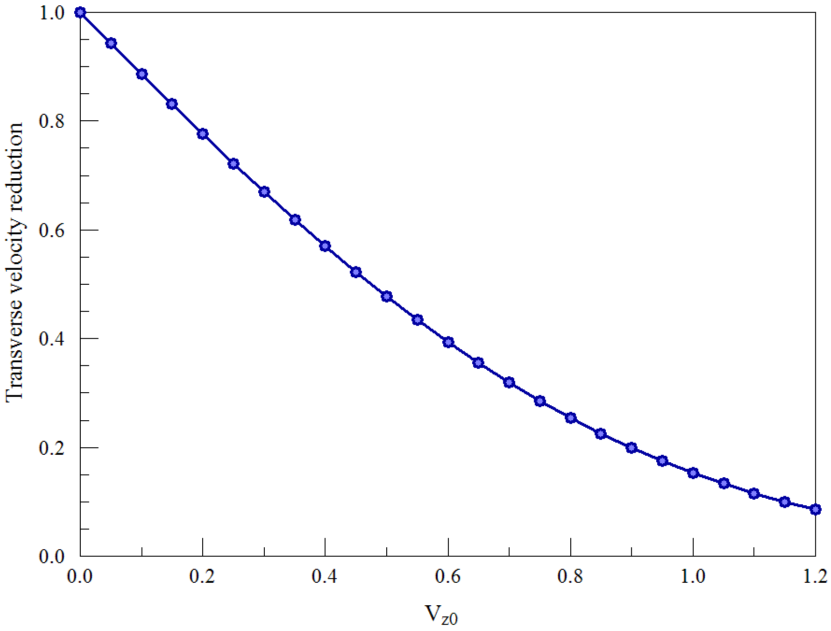

Figure 3: Reduction of the average transverse velocity of extracted electrons as a function of the dimensionless cutoff velocity.

$V_z > V_{z0}$. Dividing the second integral by the normalization integral gives a reduction factor of the average transverse velocity.

Figure 3 plots the reduction factor determined from integrals over a range of V_{z0}. The plot shows that the selection process at the emission sheath may result in significant cooling of the transverse energy distribution of the extracted beam. For example, when $j_s/j_c = 2$ the cutoff velocity is $V_{z0} \cong 0.49$ and the average transverse velocity is reduced by about 0.5. In this case, the effective source temperature in Eq. 6 is only $T_s \cong 0.25T_c$. In summary, if $j_s = j_c$ then every electron created on the cathode surface is extracted. In this case, $T_s = T_c$. When $j_s > j_c$, the effective transverse temperature is lower than the cathode temperature through velocity selection at the extraction sheath. The result is somewhat counter-intuitive. A small increase in the cathode temperature may result in a substantial reduction in the effective transverse temperature of the extracted beam.