Three-dimensional finite-element code for electrosurgery and thermal ablation simulations

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ABSTRACT

ETherm3 is a finite-element software suite for simulations of electrosurgery and RF thermal ablation processes. Program components cover the complete calculation process from mesh generation to solution analysis. The solutions employ three-dimensional conformal meshes to handle cluster probes and other asymmetric assemblies. The conformal-mesh approach is essential for high-accuracy surface integrals of net electrode currents. ETherm3 performs coupled calculations of RF electric fields in conductive dielectrics and thermal transport via dynamic solutions of the bioheat equation. The boundary-value RF field solution is updated periodically to reflect changes in material properties. ETherm3 features advanced material models with the option for arbitrary temperature variations of thermal and electrical conductivity, perfusion rate, and other quantities. The code handles irreversible changes by switching the material reference of individual elements at specified transition temperatures. ETherm3 is controlled through a versatile interpreter language to enable complex run sequences. The code can automatically maintain constant current or power, switch to different states in response to temperature or impedance information, and adjust parameters on the basis of user-supplied control functions. In this paper, we discuss the physical basis and novel features of the code suite and review application examples.

Keywords: finite-element methods, electrosurgery simulation, thermal ablation simulation RF fields, thermal transport

1. INTRODUCTION

Electrosurgery procedures[1,2] involve the application of RF electric fields to produce local heating. The goal is to kill or to alter selected tissues. Applications include blood-vessel cauterization, incision sealing and minimally-invasive procedures for tumor destruction. By their nature, electrosurgery procedures induce large changes in the thermal and electrical properties of tissue. In many procedures the application of RF power follows a complex temporal sequence to ensure effective local treatment while minimizing damage to surrounding tissues. In the past, the design of equipment and the choice of operation sequence followed entirely from empirical experience. In this case investigations of new applications can be difficult and costly. Computer simulations can play important role if closely coupled to experiments. Simulations offer a relatively quick and inexpensive approach to fill in missing data points, to investigate novel methods or to optimize existing procedures.

Considering the complexity of electrosurgery procedures, generic finite-element codes have little use. In response, we have developed specialized tools to handle the full range of material behavior and operation sequences. We previously described the ETherm programs[3–5] which perform two-dimensional simulations in cylindrical or planar geometries. We have recently completed development of ETherm3, a full three-dimensional finite-element software suite. There were several types of calculations that motivated this work:

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- Three-dimensional solutions are necessary to determine end effects for many electrosurgery and RF ablation tools.
- Solutions for simple probes may be significantly altered by the presence of nearby symmetric structures like blood vessels.
- The electric field amplitude in RF ablation procedures can be reduced with the use of arrays or multi-pronged probes.

ETherm3 uses the same command language as ETherm and runs on personal computers under Microsoft Windows or Linux. The code suite includes an interactive mesh generator, solution programs for RF fields and thermal transport and dedicated post-processors for plots and data analysis. ETherm3 has several notable features:

- Employment of conformal hexahedron meshes for high-accuracy surface integrals,
- Eoupled electrical and thermal solutions with temperture-dependent effects,
- Advanced material models,
- Built-in script interpreter with a flexible control language for autonomous operation.

The following section describes the nature of conformal hexahedron meshes and the tools used in ETherm3 for their creation. Section 3 covers solution methods for coupled RF electric fields and thermal transport. Section 4 reviews techniques for modeling temperature-dependent properties of materials (including phase changes) while Section 5 describes methods to operate probes at fixed current or as part of a temperature-control loop. Features of the intepreter language for sequence control are described in Section 6. Features of the EView3 and TView3 postprocessors are covered in Section 7. Finally, Section 8 presents examples that illustrate some advanced features of the codes.

2. MESH GENERATION

The foundation of the finite-element method for field solutions[6,7] is to divide the solution space into small volumes (*elements*). The division is performed in such a way that each element is uniquely part of one material region. This condition simplifies the mathematics as well as the organization of the code – only one material model is applied to each element. The material regions may represent electrodes, insulators or different types of tissue. The division facilitates the conversion of the governing partial differential equations for RF fields and temperature to large sets of coupled linear equations. Such systems are easily solved on a digital computer.

The process of dividing space is called *mesh generation*. Figure 1*a* shows a three-dimensional hexahedron element and the definition of some terms. The simplest way to divide space is the non-conformal mesh of Fig. 1*b* which employs box elements. The approach leads to a poor representation of surfaces between material regions. Consequently, field calculations near the surface are inaccurate. For example, surface integrals of current density (a critical operation in ETherm3) over the sphere of Fig. 1*b* may be in error by more than a factor of two. The conformal mesh of Fig. 1*c* is more difficult to construct but gives highly-proved accuracy in surface field calculations. In this mesh elements are flexed near region boundaries so that facets lie close to the desired shape. The box elements are transformed to generalized hexahedrons (Fig. 1*a*).

Mesh generation in ETherm3 performed by MetaMesh, a general utility for Field Precision thee-dimensional solution packages. The program has the unique capability to create *structured* conformal meshes. Although elements may have arbitrary shapes, their ordering follows ideal cubic logic. In other words, each element has a unique set of indices along the coordinate axes such that element $(I+1, J, K)$ is adjacent to element (I, J, K). Mesh structure allows faster field soluions, an essential feature in ETherm3 where the boundary-value solution for RF electric field may be undated hundreds of times in a run with temperature-dependent materials.

MetaMesh was originally designed to represent precision mechanical objects. For this application, the program uses constructive solid geometry techniques to fabricate complex assemblies from simple parts (*i.e.*, cylinders,

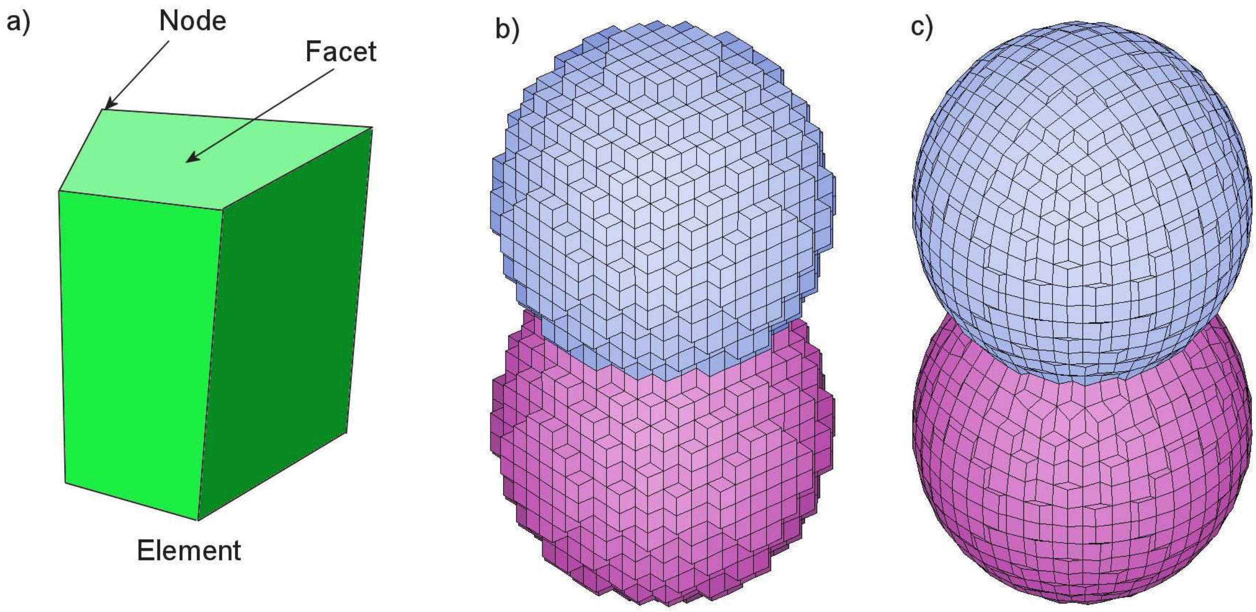

Figure 1. Three-dimensional mesh generation. *a*) Hexahedron element. *b*) Non-conformal mesh of box elements. *c*) Conformal mesh.

spheres and cones, extrusions, turnings,...). Manufacturing instructions are contained in a simple interpreter script. Figure 2 illustrates a recent code addition, the capability to replicate mathematically-generated surfaces from a table of elevation values. The mesh in the figure combines a mathematical surface with an extrusion. Table 1 shows the script to generate the shapes.

Effective modeling of biological systems requires a different approach to mesh generation. Although organs may may have extremely complex shapes, precise dimensions are usually not critical. Our main development project for MetaMesh in the coming year is to support input of three-dimensional geometries as a regular array of region numbers (voxels). After the information has been projected to a foundation mesh of box elements, the program will conformalize elements to produce smooth region surfaces. It will be possible to superimpose precision parts like probes on the same mesh with voxel-type information. The text input files to define geometries will have a simple neutral format facilitating input of several types of image data. We are also working a translation utility to convert MRI files in the Analyze format to standard MetaMesh input files.

3. FIELD SOLUTION TECHNIQUES

Electro-surgery simulations involve two dissimilar finite-element solutions: RF electric fields and thermal transport. Nonetheless, structures in the solution volume are common to both solutions. Consequently, ETherm3 uses the same mesh for the RF and thermal solutions, assigning different physical properties to the regions. Because there is little change in thermal properties over a period of the RF field, the electric field solution can be handled as a boundary value problem with periodic updating. The following equation governs the RF field[7]:

$$\nabla \cdot \left[\left(\epsilon_r - \frac{j\sigma}{\epsilon_0 \omega} \right) \nabla \phi \right] = 0. \tag{1}$$

In Eq. 1 ϵ_r is the relative dielectric constant of the medium and σ is the electrical conductivity. The quantity ϕ (the quasi-static potential) is a complex number. The amplitude and phase of the electric field is given by:

$$\mathbf{E} = -\nabla \phi. \tag{2}$$

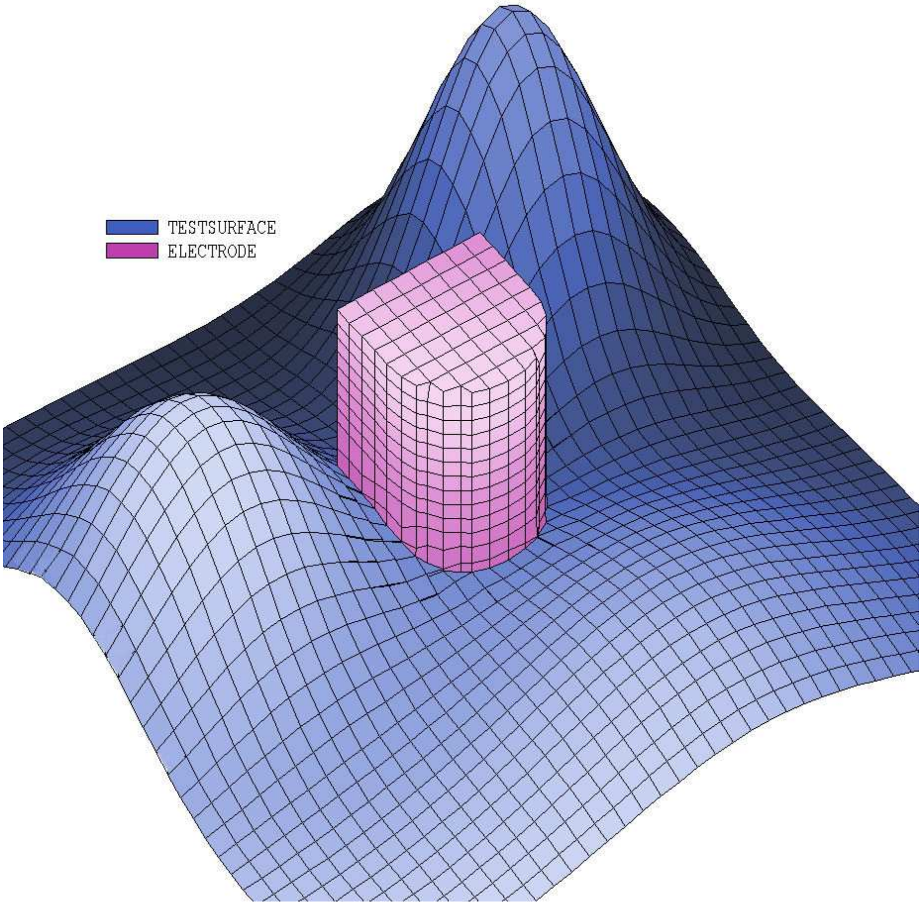

Figure 2. Plot of facets on region boudaries created by MetaMesh. The blue region is a solid with a mathematically-generated surface. The red region is a superimposed extrusion.

The electric field provides one of the source terms in the thermal transport equation:

$$q_e = \sigma \mathbf{E} \cdot \mathbf{E}^*/2. \tag{3}$$

Equations 1 and 2 include effects of both real and displacement current and hold in the *non-radiative limit*. The validity condition is that the the radiated electromagnetic energy leaving the patient is small compared to the deposited energy. The condition holds when the RF wavelength is large compared to the treatment area.

Conversion of Eq. 1 to finite-element form uses the standard minimum-residual method with quasi-linear shape functions for the hexahedron elements. The resulting linear equation set determines ϕ at the element node locations. Each equation involves coupling to 27 neighboring nodes. The coupling coefficients are computed by taking numerical integrals over elements using the normal coordinate method. The equations are solved with an iteractive matrix-inversion technique. An initial electric field solution with 400,000 nodes requires about 2 minutes on a 3.0 GHz computer. The solution time is shorter for updates with temperature-dependent ϵ_r or σ if changes are relatively small.

ETherm3 solves the following relationship (often know as the bioheat equation[8–10]) for thermal transport:

$$\rho C_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + q_e + q_m + W_b (T_b - T). \tag{4}$$

Table 1. MetaMesh script to generate Fig. 2

```
PART
  Type Surf3D FIGURE02.DAT
  Name TestSurface
  Region TestSurface
  Fab -1.00
  Surface Region SVolume 1.00
END
PART
  Type Extrusion
    L  -1.0 -1.0  1.0 -1.0 S
    L   1.0 -1.0  1.0  0.0 S
    A   1.0  0.0  0.0  1.0  0.0  0.0 S
    A   0.0  1.0 -1.0  0.0  0.0  0.0 S
    L  -1.0  0.0 -1.0 -1.0 S
  End
  Region Electrode
  Fab  9.00
  Surface Region SVolume
  Rotate 0.0 0.0 180.0
END
```

In Eq. 4 ρ is the density of the medium, C_p is the specific heat, k is the thermal conductivity, q_e is the resitive source from Eq. 3 and q_m is any other source such as metabolism. The term $W_b(T_b - T)$ is a thermal source or sink resulting from the volume-averaged perfusion of blood through capillaries. Here, W_b is the blood flow rate and T_b is the incoming blood temperature.

The finite-element solution of Eq. 4 presents some problems. Implicit methods guarantee stability but are quite difficult and time intensive to implement in three dimension solutions with millions of coupled nodes. An explicit technique like the Dufort-Frankiel method[11] (which is stable for any choice of time step) is highly desirable. Unfortunately, explicit methods are inherently unstable in the standard formulation with 27-node coupling when the mesh contains elongated elements (ratio of the maximum to minimum side lengths greater than about two). With extended node coupling, there is a close balance and cancellation of contributions of adjacent neighboring nodes resulting in a gowing oscillation of temperature between adjacent mesh layers. We corrected the problem by recasting the minimum-residual formulation for the conformal mesh to reduce coupling to the six nearest nodes. The resulting solutions are fast and stable for all mesh geometries and choices of time step at the expense of some reduction in accuracy.

An ETherm3 run consists of an initial RF solution followed by a thermal solution over a specified time span. Temperature-dependent thermal quantities are updated each time step. The electric field must be updated periodically if materials have temperature-dependent values of ϵ_r or σ. Electrical field updates may required considerable time, so it is essential to minimize operations in three-dimensional solutions with large meshes. Accordingly, the code has been carefully analyzed to eliminate unnecessary or reduntant operations. Several techniques have been implemented to reduce run time:

- Variable time steps with automatic adjustment are used in the thermal solution.
- Whenever possible, the RF solution is scaled rather than recomputed.
- For pulsed operation, the program can load precomputed baseline RF solutions.
- Element properties and coupling coefficients for the RF and thermal solutions are updated only for temperature-dependent materials.

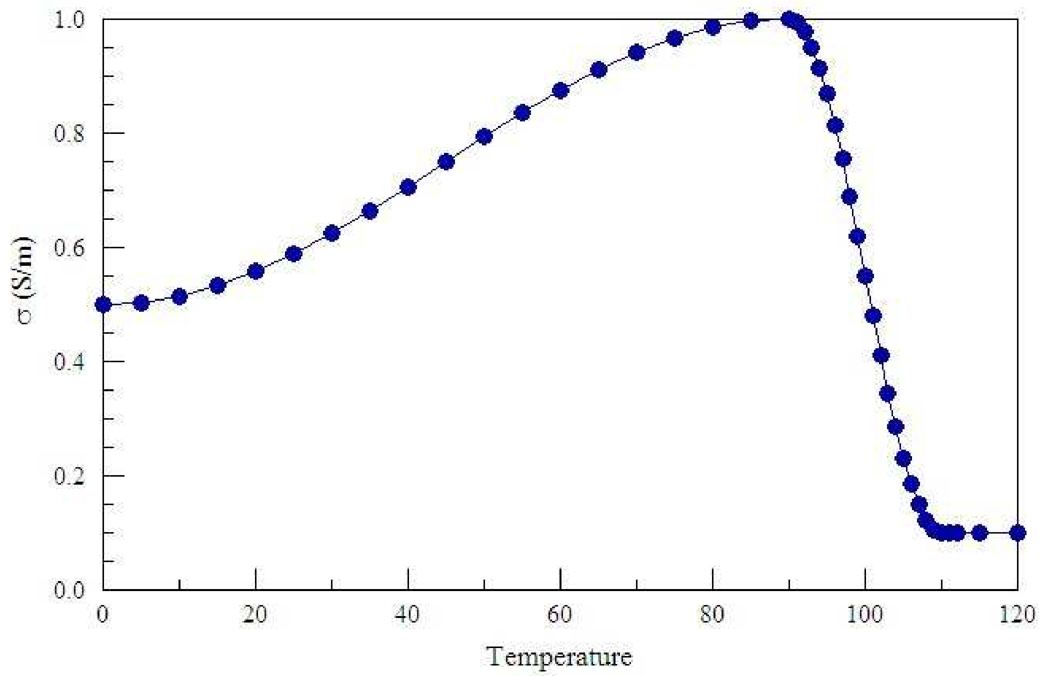

Figure 3. Tabular function to define a temperature-dependent electrical conductivity. Note how values are clustered near the sharp transition at 100^o.

4. MATERIAL MODELS AND TEMPORAL VARIATIONS

Interesting electrosurgery simulations usually involve materials with significant variations of properties with temperature. Parametric material models lack versatility and extensibility. Therfore, ETherm3 uses a flexible method of *tabular functions* to represent arbitrary temperature and time variations. A table is simply a text file where each line contains one value of an independent and dependent variable (such as temperature T and thermal conductivity $k(T)$). Figure 3shows a typical data set to represent a temperature-dependent electrical conductivity. ETherm3 incorporates a standard software unit used in all Field Precision programs to perform cubic spline or linear interpolations. The unit is designed so that tables appear as a continuous function to the calling program. The routines handle non-uniform intervals of the dependent variable. Flexibility is achieved through a free-form parser with automatic sorting and error checking. Tabular input has several advantages for the user, including data transparency, the option to maintain standard material libraries, efficient input of digitized or published data and easy electronic exchange of models.

Fixed values or tables may be applied to the following quantities in Eqs. 1 and 4: σ, ϵ_r, C_p, k and W_b. Physical quantities are assigned to *materials*. The program accepts up to 127 different material types. The elements of the mesh are divided into *regions* that represent physical objects like tissue types, dielectrics or electrodes. In turn, an element with a given region number is assigned an initial material and may be associated with different materials, depending on its thermal history. Flexible material assignment is the key to the implementation of phase changes. The properties of a region represented by a single table are non-linear but reversible. If the tissue cools, it returns to its original state. Etherm3 represents permanent state changes by ressigning the material reference when the element reaches a setpoint temperature. ETherm3 also maintains values of the Arrhenius damage integral that represent the extent of chemical changes. Damage integral values can also be used as setpoints. With the reassignment method properties can be permanently changed, even if the element cools. For example, suppose an element is originally associated with a material with electrical conductivity that drops sharply at 100 degrees. At this temperature, the reference is shifted to a material with the properties of dessicated tissue. The conductivity will not rise even if the electrical power and tempertaure drops. All elements of a region have the same region number but may have different material associations. A region may have up to five alternate materials with ascending temperature or damage-integral setpoints.

5. FIXED-CURRENT OPERATION AND TEMPERATURE CONTROL

As mentioned in Sect. 3, the RF solution yields the quasi-static potential at all points in solution volume for specified electrode potentials. In many applications the equipment operates at fixed current or power rather than voltage. These global quantities are not natural boundary conditions of Eq. 1, so we must use an indirect approach in ETherm3. The current or power is calculated using the present voltages and material states. An adjustment factor to achieve the desired state is calculated and used to scale potential values.

The global power P_0 is relatively easy to calculate by taking a volume integral of time-averaged resistive power density ($\sigma \mathbf{E}\mathbf{E}^*/2$) over all material elements of the solution volume. Adjustment of a specific voltage in a multi-electrode systems is an undefined procedure – there are an infinite number of possibilities. Therefore, we assign a general amplitude change to all electrodes. If P is the desired power level, then potential values are scaled by $\sqrt{P/P_o}$. The calculation of current from an electrode requires a surface integral of current density over the fixed-potential region. To start, ETherm3 gathers a list of facets of fixed-potential region elements that are common to conductive/dielectric elements. The program determines the current from each facet using a 4×4 normal coordinate integral of electric field, and sums the real and displacement current to find the amplitude I_0. If I is the desired current amplitude, potential values at all nodes are scaled by I/I_0.

The tabular functions described in Sect. 4 are also employed in ETherm3 to represent arbitrary temporal variations such as a time-dependent thermal source q_m or boundary temperature. The program can also store several generalized functions that can be used for feedback control. For example, suppose we create a function that equals unity from 0.0 to 60.0 and then drops sharply. In this case, we could use a region-averaged temperature as the independent variable and use the tabular function output as a multiplying factor for probe voltages. For a good choice of control function, ETherm3 would generate a programed voltage that would maintain average temperature in a region near 60.0 ^{o}C. The control function capability makes ETherm3 a useful tool to test the dynamics and stability of control loops in electrosurgery equipment.

6. SEQUENCE CONTROL

Because of wide variety of operating modes encountered in electrosurgery and RF ablation procedures, we have equipped ETherm3 with a versatile control capability based on a custom interpreter language. The program can make decisions and change operating modes depending on the temperature history of the medium. The organization of the ETherm3 control script follows the logic of an actual treatment. Initially there are given conditions that are not controlled by the operator: the geometry of the patient and probe and the intrinsic properties of materials (including temperature dependencies). During the procedure, the operator can modify applied voltages and the pulse sequence in response to changes of electrical and thermal properties. Accordingly, the ETherm3 control script contains one SETUP section for the definition of general control parameters and material properites. This section is followed by an unlimited set of STATE sections. Each STATE section represents a particular activity that the operator could take, such as "apply 2000 mA current until the probe impedance rises 10% from its initial value," or "drop the voltage to half its present value until the maximum temperature of the region around the drops to less than 60 degC."

There is insuffcent space in this paper to describe the ETherm3 control language in detail. In summary, STATE commands perform the following types of activities:

- Set STATE properties such as the maximum duration and frequency of command updates.
- Set parameters of the RF solution such as voltage amplitude and phase, RF frequency or global power.
- Return values of state variables such as average temperature in a region, elapsed time in a STATE, and electrode current.
- Store numbers in program variables with the operation to perform a broad range of mathematical operations.

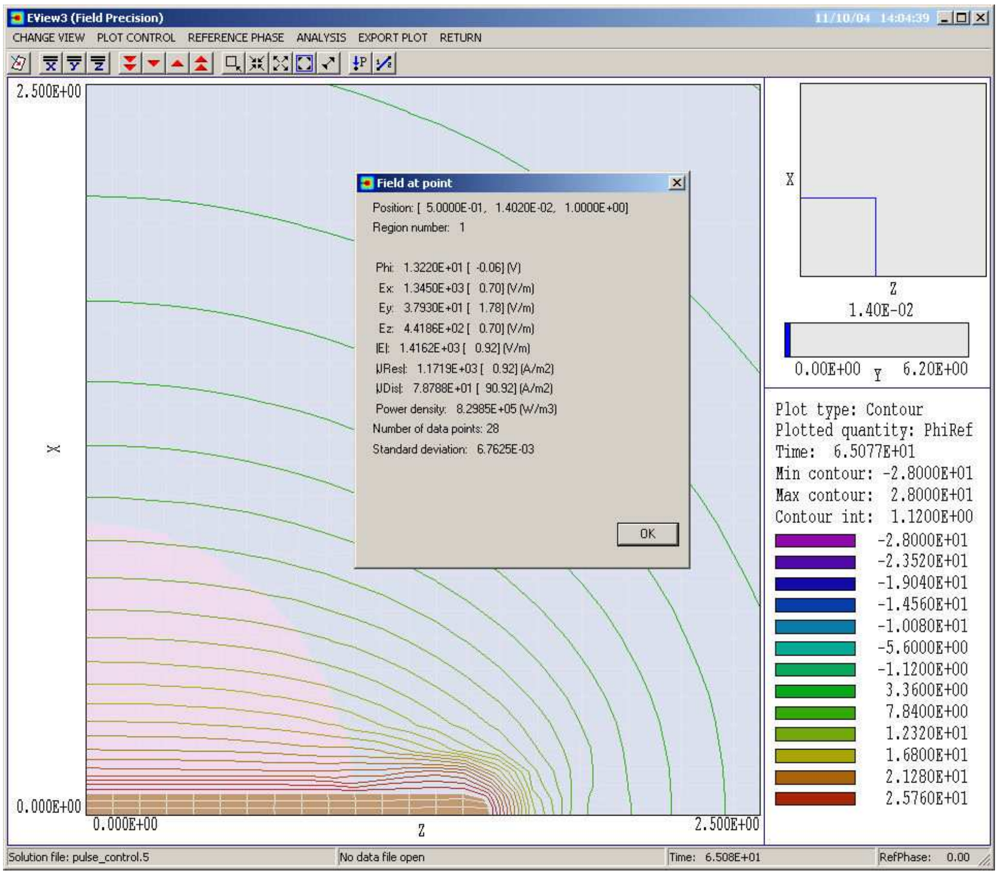

Figure 4. Screen shot of the EView3 post-processor for RF electric fields illustrating analysis capabilities in the two-dimensional slice plot mode.

- Test conditions to exit a STATE.
- Follow control loops to repeat a group of STATES.

Table 2 shows an example of the control language.

ETherm3 can record a variety of diagnostic data during the thermal solution. The program can create data files (snapshots of the RF or thermal solutions) at up to 500 specified times. The user can place up to 20 diagnostic probes at different positions in the solution volume. The probes generate output files containing complete records of temporal variations of electric and thermal quantities.

7. ANALYSIS TOOLS

The ETherm3 code suite includes EView3 and TView3, dedicated post-processors to generate plots and to analyze RF and thermal solutions. The programs feature interactive graphical capabilities that allow the user to move easily through the solution volume. Two dimensional plots show quantities in a plane normal to the x, y or z axes. There are two styles. The *plane plot* is a projection of field quantities to a rectangular grid. The simple grid makes it possible to construct advanced displays such as 2D and 3D filled-contour plots. The slice plot is a projection that preserves the structure of the conformal mesh. This mode accurately shows regions boundaries and allows point-and-click display of point or scan quantities in the plane (Fig. 5). RF solution quantities that can be displayed include the quasi-static potential, electric field components, real and displacement currents and power density. The code can show either the amplitude of field quantities or a snapshot of the field at a

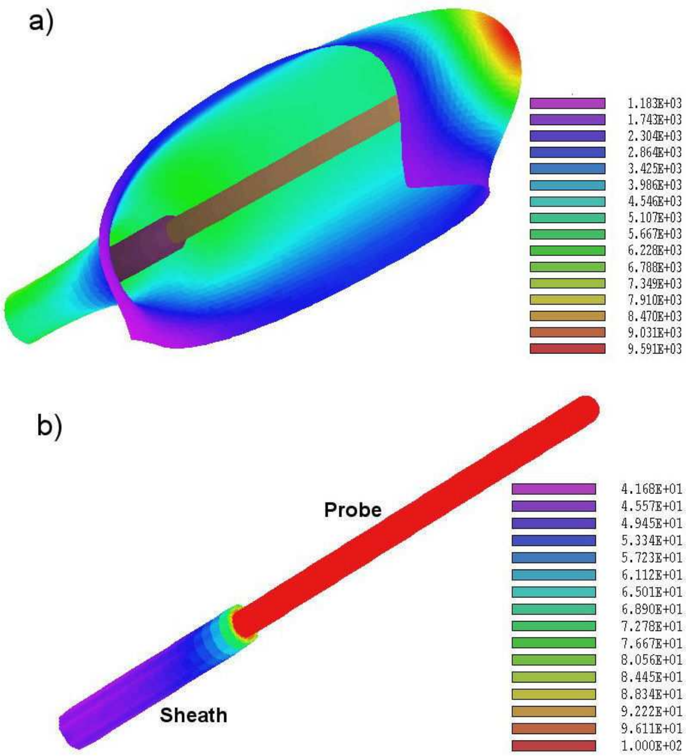

Figure 5. Available three-dimensional plot types in TView3. *a*) Isotherm surface with color-coding determined by the magnitude of thermal flux. *b*) Region boundaries of the probe and sheath color-coded by temperature.

given reference phase. EView3 can also plot spatial variations of $\epsilon_r(T)$ and $\sigma(T)$ for solutions with temperature-dependent materials. Thermal quantities plotted by TView3 include the temperature and thermal flux. Spatial variations in $k(T)$, $C_p(T)$ and $W_b(T)$ can be displayed for solutions with temperature-dependent materials.

Both programs include advanced 3D plots with surfaces constructed directly from the conformal mesh. Figure 5 shows two examples from TView3. The top plot shows an isothermal surface with color-coded by thermal flux. This type of plot is useful for displaying the shape of the treatment volume. The bottom plot is a representation of boundary facets on seclected regions with color-coding by temperature. This plot type is useful to display surface temperatures on electrodes and organs. By splitting objects into two or more regions in MetaMesh, it is possible to display temperature variations on arbitrary three-dimensional surfaces.

Both post-processors have extensive capabilities for quantitative analysis. These include point calculations and line scans using a second-order, least-squares-fit procedure. Information can be displayed within the program or ported to an analysis history file in text format. Line scans generate screen plots with digital ocilloscope features. Complex or repetitive operations can be controlled by scripts. The programs can perform automatic volume integrals over regions of the solution space. Calculated quantities include RF field energy, total resistive power dissipation, and average temperature. Surface integrals employ the same routines used in ETherm3 for current calculations. Output quantities include net current or thermal flux. The post-processors include the option to create matrix files where electrical or thermal quantities are projected from the conformal mesh to regular box mesh. This information can easily be ported to user programs or other analysis software.

8. APPLICATION EXAMPLES

This section reviews two examples that illustrate some of the features of ETherm3. This first addresses solution accuracy, operation at constant current and the ability to handle temperature-dependent materials. The simple test geometry consists of concentric cooled spherical electrodes of radii $r_i = 1.0$ cm and $r_o = 5.0$ cm seperated

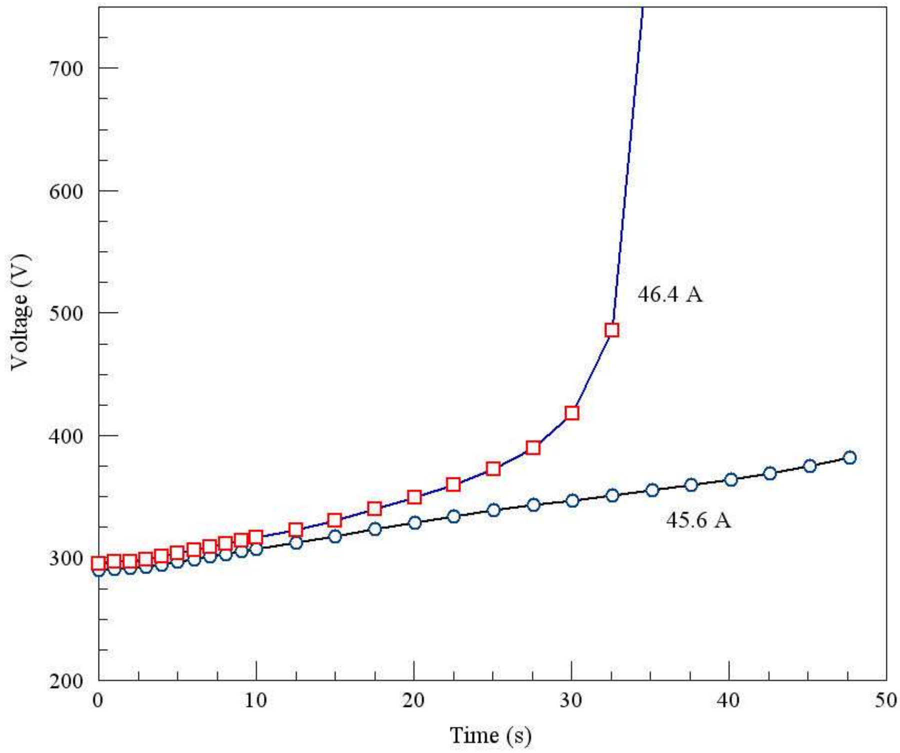

Figure 6. Variation of voltage with time for drive currents of 5.7 and 5.8 A, first application example

by a uniform medium with $\epsilon_r = 1.0$. The electrical conductivity is defined by a table with values that start at $\sigma = 1.0$ S/m at $T = 0.0^o$C and drop smoothly to 0.2 S/m in the range $T = 20.0^o$C to $T = 120.0^o$C. Both electrodes are maintained at a reference temperature of 0.0 ^{o}C. The outer electrode is grounded and the inner electrode has a time-dependent voltage to maintain constant current. The medium has the thermal properties $k = 100.0$ W/m-^{o}C, $C_p = 4800.0$ J/kg-^{o}C and $\rho = 1000.0$ kg/m^3. The approximate side length of the elements near the inner sphere is 0.1 cm.

The total current flowing through the medium is determined by a surface integral of normal electric field over the inner sphere. Initially all elements are near 0.0 ^{o}C and the conductivity is $\sigma \cong 1.0$ S/m. We can compare the impedance calculated by the code to the analytic formula:

$$R = \frac{\left(\frac{1}{r_i} - \frac{1}{r_o}\right)}{4\pi\sigma}. \quad (5)$$

Inserting values in Eq. 5 gives 6.366 Ω, while the ETherm3 calculation yields 6.371 Ω. The small difference results from the fact that the shape used by ETherm is a multi-facteted polyhedron rather than a sphere.

The thermal solution advances for 50 s with a variable time step. The control script contains three STATES. The first sets up a baseline RF solution at $f = 60.0$ Hz with 250 V applied to inner sphere. The second STATE advances the thermal solution for 10 seconds, updating commands at 1 second intervals to include effects of changing electrical conductivity. The STATE includes the command

```
Current(3) = 5.7
```

The command maintains approximately constant current from the inner sphere (Region 3 in the solution space) by correcting the voltage. In response to the command, ETherm3 recalculates the field every 1.0 second, takes the surface integral to determine the current, and scales potential values in the solution space to bring the current

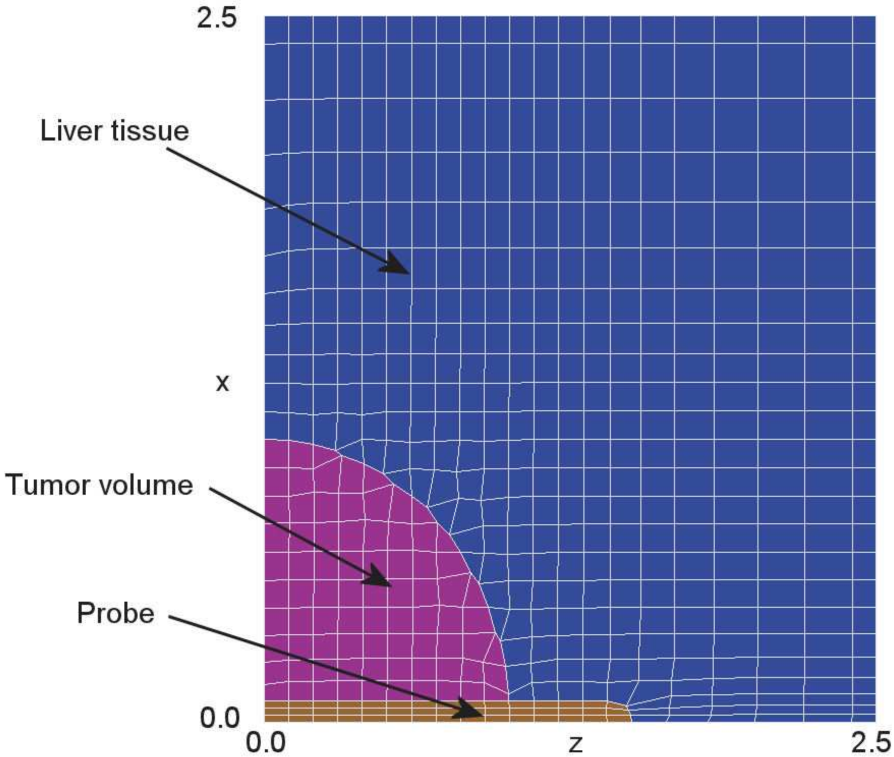

Figure 7. Mesh representation of the geometry of the second application example, slice in the plane $y = 0.0$.

to 5.7 A. Additional commands instruct the program to record values of voltage and maximum temperature in the solution volume at each update. The third state (which runs from 10 to 50 s) has the same commands with an update interval of 2.5 seconds. Figure 6 shows the voltage variation required to maintain constant current with decreasing conductivity for two values of the drive current. The 5.8 A curve exhibits thermal run-away. A constant-current solution can become unstable when the electrical conductivity drops with increasing temperature. Thermal run-away is a real effect rather than a numerical artifact. We have observed similar phenomena in experiments with electrosurgery equipment. Figure 6 demonstrate that non-linear solutions much be approached with caution – a small change in input can make a dramatic difference in output.

Figure 7 shows the geometry of the second application example. Hee we consider a thin probe (of the type used in RF ablation therapy) with length 3.0 cm inserted in a 2.0 cm diameter tumor. The cylindrically-symmetric geometry was chosen to allow comparisons with the two-dimensional ETherm code. The calculation was performed in single octant with symmetry boundaries at $x = 0.0$, $y = 0.0$ and $z = 0.0$ using a mesh of 74,000 elements. In the calculation, both the liver and tumor have the conductivity variation shown in Fig. 3. Electrical onductivity rises 2% per degree up to around 100 degC where a sharp drop represents tissue vaporization.

Table 2 shows the main portion STATE sections of the control script. The commands included from the initial STATE store a value of applied voltage in program variable X(4) as a parameter, calculate a baseline normalized solution to determine the zero-temperature impedance and set a trigger level 15% higher. The next three STATES are contained in a loop that runs for a maximum time of 402 seconds. The commands of the first STATE setr the voltage to X(4) and advance the thermal solution until either 1) the state time reaches 200 s or 2) the impdedance exceeds the trigger level. The RF solution is updated and checks are performed at 2.5 second intervals. The second STATE has zero duration so the commands are executed only once per loop cycle. No action is taken if the duration of the previous state exceeds 10 s. Otherwise, the baseline voltage is reduced by 1.0 V. The commands of the third STATE reduced the voltage to 2.0 V and advance the thermal solution for 15 seconds to allow cooling of the region near the probe.

Table 2. ETherm3 script to generate application example 2

```
    ...
      X(4) = 28.0
      Relax
      X(10) = Impedance(3)
      X(2) = X(10) 1.15 *
    ENDSTATE
  #STARTLOOP
    STATE 200.0 2.5
      Voltage(3) = X(4)
      Relax
      Show Impedance(3)
      Show MaxTemp
      X(3) = StateTime
      X(1) = Impedance(3)
      Test X(1) > X(2)
    ENDSTATE
    STATE 0.0 0.0
      Test X(3) > 10.0
      X(5) = X(4) 1.0 -
      X(4) = X(5)
    ENDSTATE
    STATE 15.0 15.0
      Voltage(3) = 2.0
      Relax
    ENDSTATE
  #GOTO STARTLOOP 402.0
```

Figure 8 shows the history of probe impedance and maximum temperature in the solution volume. In the first pulse cycle, the impedance initially drops as the medium heats and then follows a rapid rise as the medium around the probe tip approaches 100 ^{o}C. Figure 9 shows the spatial distribution of electrical conductivity in the plane $y = 0.0$ late in the pulse (t = 65.1 seconds). Although some areas have enhanced conductivity, σ is strongly sharply reduced near the probe tip (lower-left). The spatial variation is quite complex and the problem would be imposible to treat with analytic models. In the impedance plot, triggering does not occur exactly at the setpoint level of 300 Ω because checks are performed only at 2.5 second intervals. The lower plot shows the maximum temperature recorded in the solution space (at a point near the tip of the probe in the region of enhanced electric field). Records of temperature at other positions confirm that the pulse cycle confines the bulk of medium surrounding the probe to a temperature less than 100 ^{o}C. The simulation involved 120 recalculations of the electrical field with a total run time of 22 minutes on a 3 GHz computer. The example illustrates that complex three-dimensional simulations are feasible on standard personal computers with careful pre-analysis to eliminate unnecessary details.

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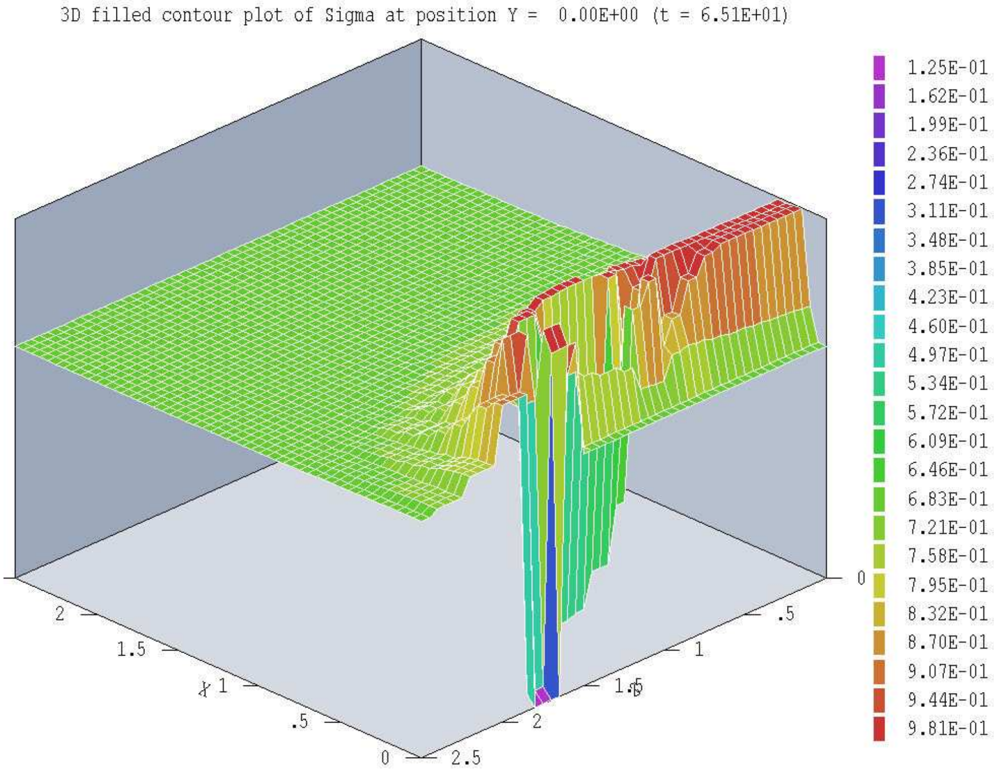

Figure 8. Temporal variations, second application example. a) Probe impedance. b) Maximum temperature in solution volume

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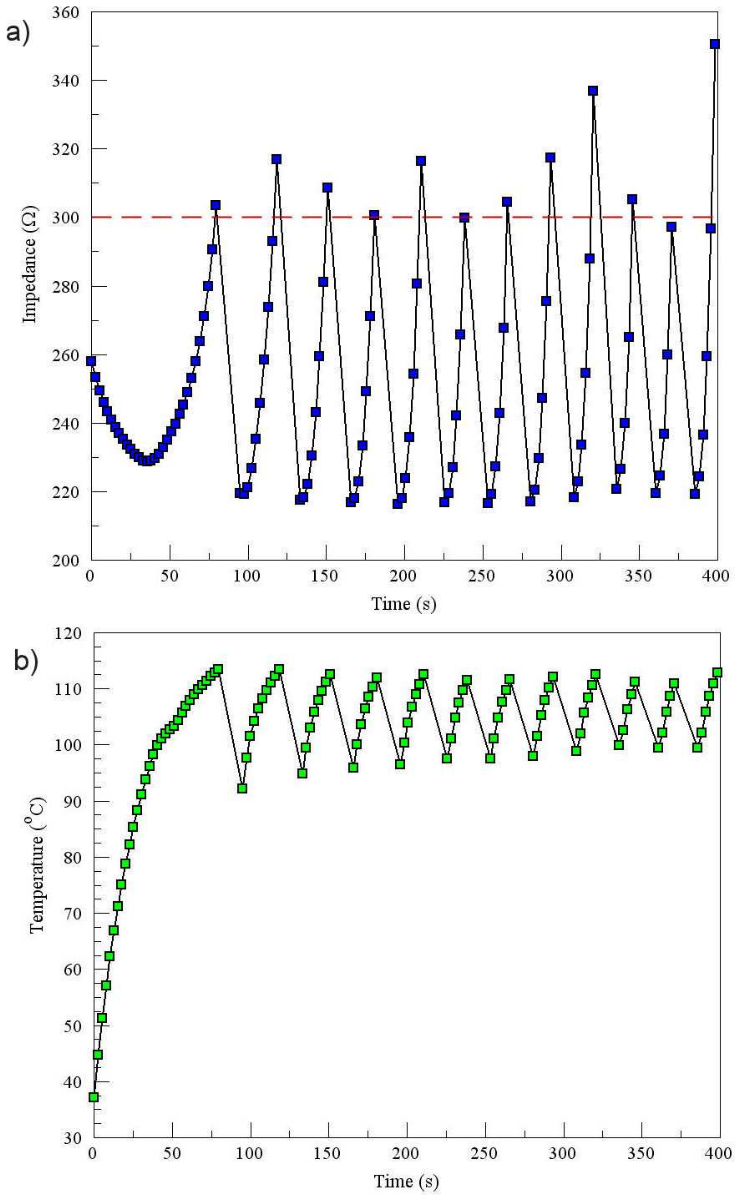

Figure 9. Spatial variation of electrical conductivity in the plane $y = 0.0$ at $t = 65.1$ s, second application example