



Tutorial: magnetic dipole moment of a coil assembly

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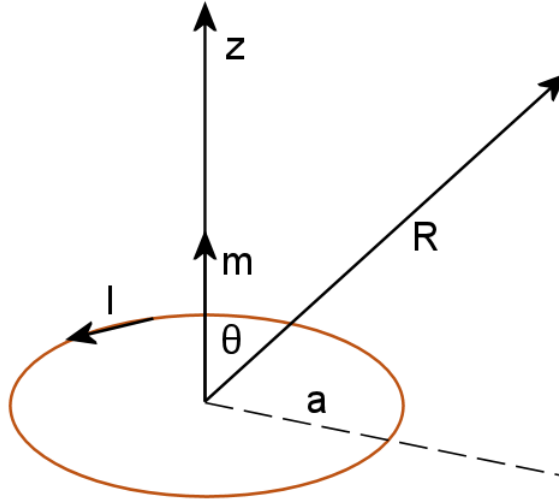


Figure 1: Magnetic dipole moment vector of a circular current loop of radius a . The quantity R is the distance from the center of the loop and θ is the angle with respect to the z axis.

Sometimes we need to find small magnetic fields at a relatively large distance from a magnet assembly. One application might be to determine shielding requirements for sensitive equipment in an MRI installation. Numerical field solutions are performed in a finite volume. The problem is that the common boundary conditions for magnetostatic finite-element solutions strongly influence weak fields at large distances, leading to inaccuracies. This tutorial illustrates how to make good estimates of distant fields.

The far-field variation of most magnet assemblies approaches that of a simple dipole. The flux density of a dipole is given in spherical coordinates by:

$$\mathbf{B} = \left(\frac{\mu_0 m}{4\pi R^3} \right) (\mathbf{a}_R 2 \cos \theta + \mathbf{a}_\theta \sin \theta). \quad (1)$$

The quantity m is the magnitude of the magnetic moment of the assembly with units of A-m². Figure 1 shows the definition of the coordinate system with respect to the direction of the vector magnetic moment \mathbf{m} . The magnetic moment of the single-turn loop shown in the figure with current I and radius a is

$$\mathbf{m} = \mathbf{a}_z I \pi a^2. \quad (2)$$

To make an accurate estimate of the flux density at a distance from a large magnet, we can use the following procedure:

- Set up a solution with boundaries that extend to the far-field region. Find calculated values of flux density at positions along the z axis ($\theta = 0.0^\circ$) that are removed from both the assembly and the boundaries. Then, fit the values to the $1/R^3$ variation of Eq. 1 to find m .
- Use the value of m in Eq. 1 to predict the magnetic flux density at positions near or outside the boundaries.

A two-dimensional cylindrical solution illustrates the method and demonstrates accuracy limits set by boundaries (**MAGDIPOLE**). Figure 2 shows one half of a cylindrical coil. The solution is performed in the space $z \geq 0.0$ m with a Neumann boundary at $z = 0.0$ m. The full coil has inner radius 0.04 m, outer radius 0.05 m, length 0.10 m and current $I = 1000$ A-turn. For comparison, the magnetic moment of a circular loop with radius $a = 0.045$ m and current $I = 1000$ A is 6.362 A-m². The coil encloses a region of radius 0.03 m and full length 0.14 m. The magnetic permeability of this region may be set to that of air ($\mu_r = 1.0$) or ferrite ($\mu_r = 500.0$). The boundaries of the solution volume are $z_{max} = 0.40$ m and $r_{max} = 0.40$.

In a cylindrical **PerMag** solution, the axis always assumes the Dirichlet condition $rA_\theta = 0.0$ tesla-m². We can choose a Dirichlet or Neumann condition for the outer boundaries at the top and the right-hand side. The Dirichlet condition (top plot in Fig. 2) corresponds to a perfectly-conducting wall. Lines of \mathbf{B} that would extend into free space are confined within the solution cylinder. The effect is to reduce the value of B_z along the z axis close to the right-hand wall. The Neumann boundary condition (bottom of Fig. 2) constrains lines of \mathbf{B} to be normal to the boundary. The condition has the opposite effect to that of the Dirichlet condition. It increases the value of B_z along the z axis near the wall. An inspection of the figure shows that the fields of the two solutions near the assembly are the same, but there are large differences near the boundaries.

Figure 3 shows quantitative results for the two boundary conditions. The value of m was calculated from a scan of B_z along the z axis from $z = 0.10$ m to the boundary according to:

$$m = \left(\frac{2\pi}{\mu_0} \right) B_z(z, 0) z^3. \quad (3)$$

The plots show computed values of m for the two boundary types without (top) and with (bottom) the ferrite slug. For comparison, the dashed red line gives the value for a circular loop with $I = 1000$ A and $a = 0.045$ m.

Although the numerical values diverge significantly from the ideal free-space result, they contain information sufficient to make a good estimate of the far fields. First, consider the plot for the air region. The quantity $B_z(z, 0)$ approaches the far-field variation at $z = 0.12$ m (a distance approximately

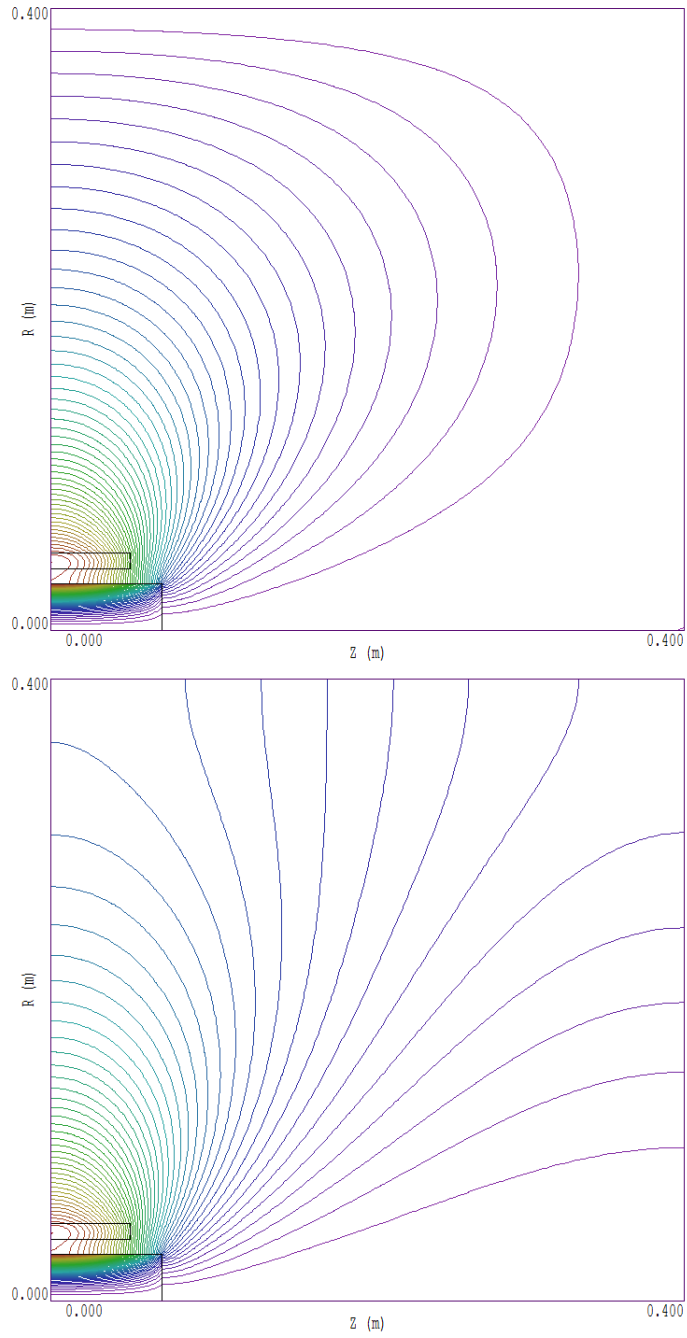


Figure 2: Lines of magnetic flux density generated by a cylindrical coil enclosing a ferrite slug. Top: Dirichlet boundary (perfectly-conducting wall). Bottom: Neumann boundary.

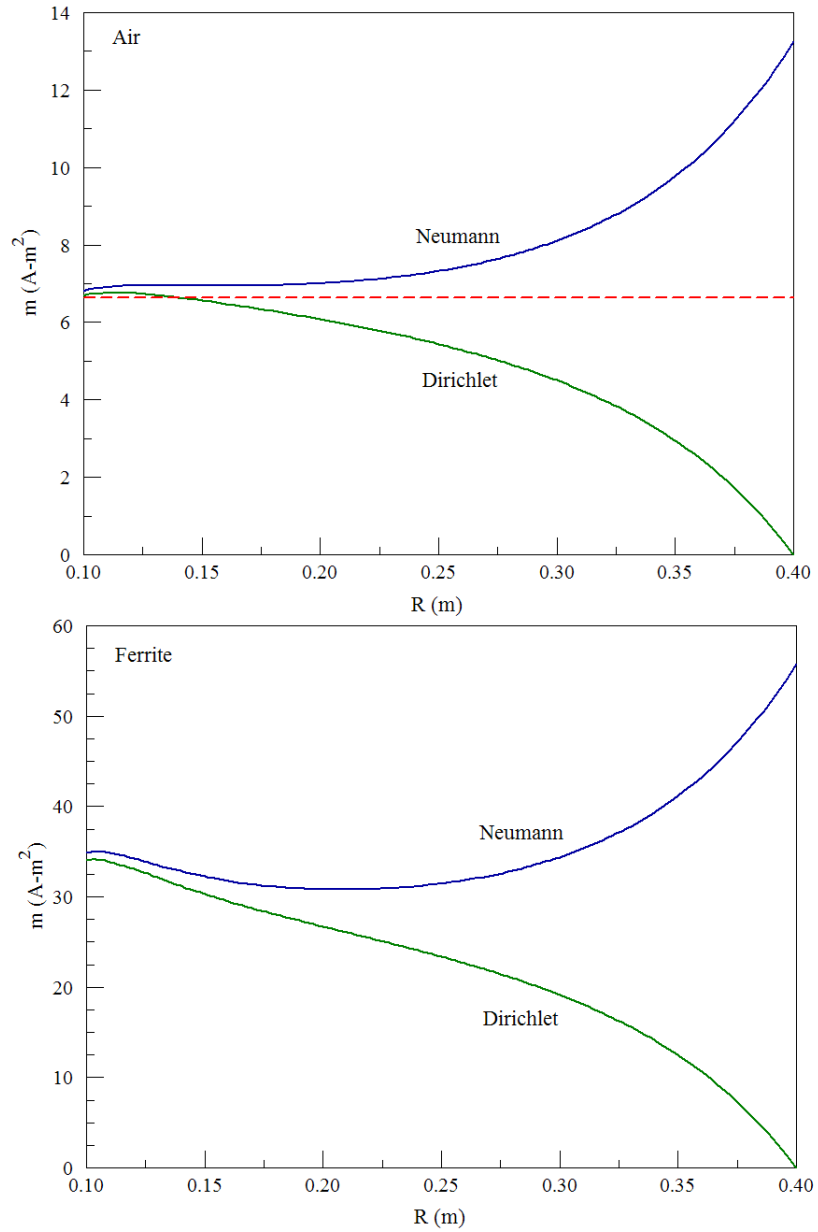


Figure 3: Variation of calculated magnetic moment m along the z axis with Dirichlet and Neumann boundary conditions. Top: Enclosed region has $\mu_r = 1.0$. Dashed red line is the theoretical value for a thin loop. Bottom: Enclosed region has $\mu_r = 500.0$.

equal to the outer diameter of the coil). With a Dirichlet boundary, the calculated magnetic moment remains constant to about $z = 0.2$ m and then rises because of the boundary effect. The Dirichlet boundary has a stronger influence, reducing B_z to zero at the boundary. The value of m at $z = 0.18$ m with the Neumann boundary is slightly higher than the theoretical value for an equivalent circular loop. The difference is real, resulting from the non-zero length and radial thickness of the coil. An inspection of the graph shows that $m \cong 7.0$ A-m².

The curves in the bottom plot of Fig. 3 settle to the far-field variation at larger z because of the protruding ferrite object. An inspection of the curves yields a value of 30 A-m² at $z = 0.2$ m, probably accurate to a few percent. The numerical calculation yields a good answer rather than a perfect answer. The following conclusions gives some perspective on the analysis:

- It would be quite difficult to derive the magnetic moment of the ferrite assembly with analytic methods.
- Given the value of m , we can find distant fields to good accuracy using Eq. 1. In contrast, the direct field values in the numerical solutions near the boundary differ substantially from the free-space dipole variation.
- The numerical solution shows that the ferrite slug increases the magnetic moment for given coil parameters more than a factor of four. The amplification is significant for applications such as satellite torque generators.